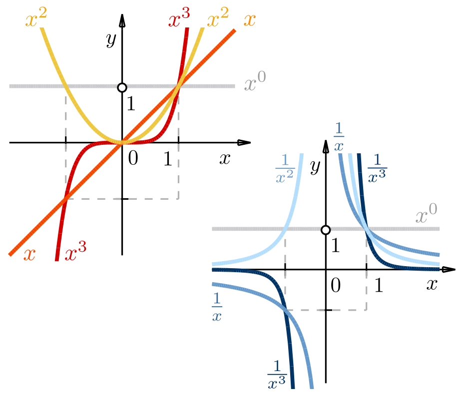
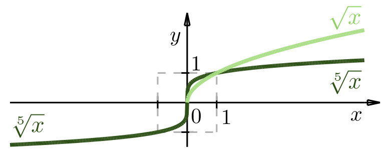
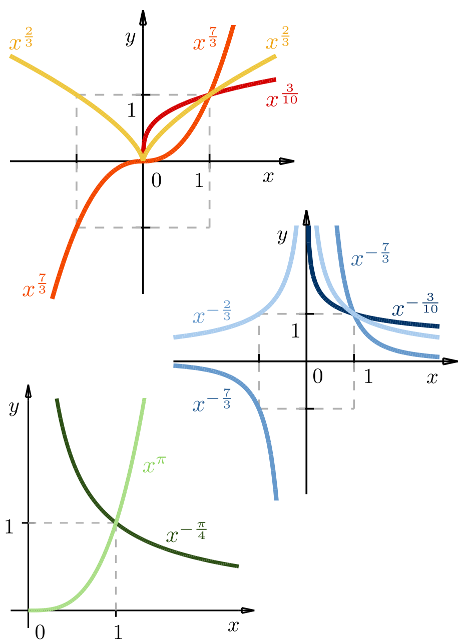
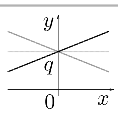
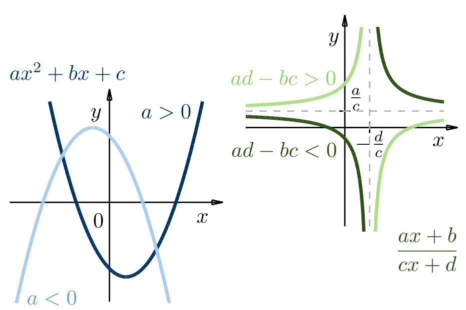
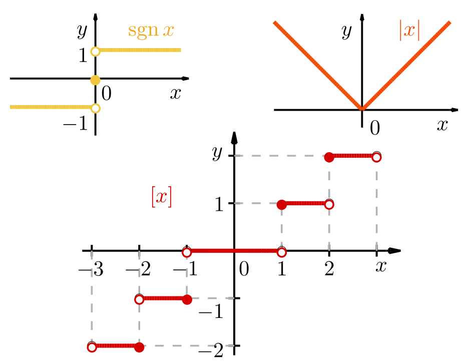
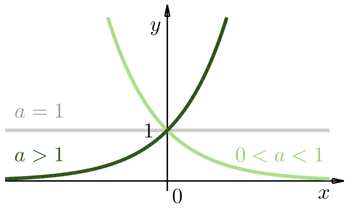
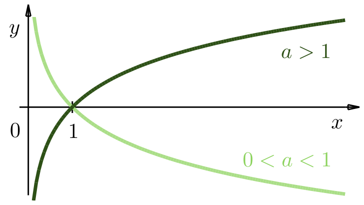
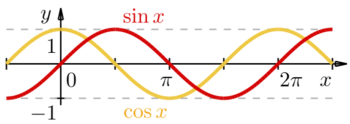
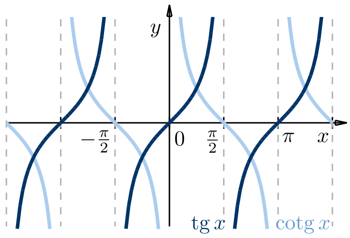
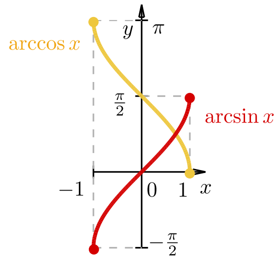
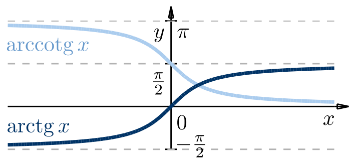
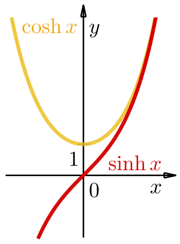
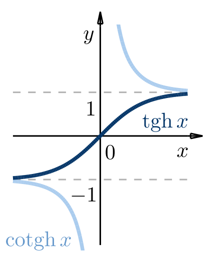
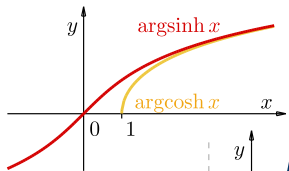
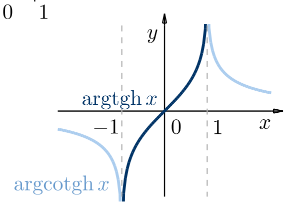
# Reálné funkce jedné reálné proměnné

| | | | | $D(f)$ | $H(f)$ | pozn. | graf funkce $f$ | $D(f')$ | derivace $f'$ |
|---|---|---|---|---|---|---|---|---|---|
| mocninná funkce s celým exponentem | $f\colon y = x^0$ | | | $\mathbf{R}\setminus\{0\}$ | $\{1\}$ | sudá |  | $\mathbf{R}\setminus\{0\}$ | $f'(x) = 0$ |
| | $f\colon y = x^n$ | $n \in \mathbf{N}$ | $n$ sudé | $\mathbf{R}$ | $\langle 0, +\infty)$ | sudá | | $\mathbf{R}$ | $f'(x) = nx^{n-1}$ |
| | | | $n$ liché | $\mathbf{R}$ | $\mathbf{R}$ | lichá | | $\mathbf{R}$ | |
| | $f\colon y = x^{-n}$ | $n \in \mathbf{N}$ | $n$ sudé | $\mathbf{R}\setminus\{0\}$ | $(0, +\infty)$ | sudá | | $\mathbf{R}\setminus\{0\}$ | $f'(x) = -nx^{-n-1}$ |
| | | | $n$ liché | $\mathbf{R}\setminus\{0\}$ | $\mathbf{R}\setminus\{0\}$ | lichá | | $\mathbf{R}\setminus\{0\}$ | |
| n-tá odmocnina | $f\colon y = \sqrt[n]{x}$ | $n \in \mathbf{N}$ | $n$ sudé | $\langle 0, +\infty)$ | $\langle 0, +\infty)$ | |  | $(0, +\infty)$ | $f'(x) = \frac{1}{n}x^{\frac{1}{n}-1}$ |
| | | | $n$ liché | $\mathbf{R}$ | $\mathbf{R}$ | lichá | | $\mathbf{R}\setminus\{0\}$ | |
| mocninná funkce s obecným exponentem | $f\colon y = x^{\frac{m}{n}}$ | $m, n \in \mathbf{N}$ | $m$ sudé, $n$ liché | $\mathbf{R}$ | $\langle 0, +\infty)$ | sudá |  | $\mathbf{R}\setminus\{0\}$ ($\mathbf{R}$ pro $m \geq n$) | $f'(x) = \frac{m}{n}x^{\frac{m}{n}-1}$ |
| | | | $m$ liché, $n$ liché | $\mathbf{R}$ | $\mathbf{R}$ | lichá | | $\mathbf{R}\setminus\{0\}$ ($\mathbf{R}$ pro $m \geq n$) | |
| | | | $m$ liché, $n$ sudé | $\langle 0, +\infty)$ | $\langle 0, +\infty)$ | | | $(0, +\infty)$ | |
| | $f\colon y = x^{-\frac{m}{n}}$ | $m, n \in \mathbf{N}$ | $m$ sudé, $n$ liché | $\mathbf{R}\setminus\{0\}$ | $(0, +\infty)$ | sudá | | $\mathbf{R}\setminus\{0\}$ | $f'(x) = -\frac{m}{n}x^{-\frac{m}{n}-1}$ |
| | | | $m$ liché, $n$ liché | $\mathbf{R}\setminus\{0\}$ | $\mathbf{R}\setminus\{0\}$ | lichá | | $\mathbf{R}\setminus\{0\}$ | |
| | | | $m$ liché, $n$ sudé | $(0, +\infty)$ | $(0, +\infty)$ | | | $(0, +\infty)$ | |
| | $f\colon y = x^a$ | $a \in \mathbf{R}\setminus\mathbf{Q}$ | $a > 0$ | $\langle 0, +\infty)$ | $\langle 0, +\infty)$ | | | $(0, +\infty)$ | $f'(x) = ax^{a-1}$ |
| | | | $a < 0$ | $(0, +\infty)$ | $(0, +\infty)$ | | | $(0, +\infty)$ | |
| | $f\colon y = q$ |  $k > 0$, $k = 0$, $k < 0$ | | $\mathbf{R}$ | $\{q\}$ | |  | $\mathbf{R}$ | $f'(x) = 0$ |
| | $f\colon y = kx + q$ | $k \neq 0$ | | $\mathbf{R}$ | $\mathbf{R}$ | | | $\mathbf{R}$ | $f'(x) = k$ |
| | $f\colon y = ax^2 + bx + c$ | | | $\mathbf{R}$ | | | | $\mathbf{R}$ | $f'(x) = 2ax + b$ |
| | $f\colon y = \dfrac{ax+b}{cx+d}$ | $c \neq 0$, $bc - ad \neq 0$ | | $\mathbf{R}\setminus\{-\frac{d}{c}\}$ | $\mathbf{R}\setminus\{\frac{a}{c}\}$ | | | $\mathbf{R}\setminus\{-\frac{d}{c}\}$ | $f'(x) = \dfrac{ad-bc}{(cx+d)^2}$ |
| | $f\colon y = \operatorname{sgn} x = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0. \end{cases}$ | | | $\mathbf{R}$ | $\{-1, 0, 1\}$ | lichá |  | $\mathbf{R}\setminus\{0\}$ | $f'(x) = 0$ |
| | $f\colon y = \lvert x\rvert = \begin{cases} x, & x \geq 0, \\ -x, & x < 0. \end{cases}$ | | | $\mathbf{R}$ | $\langle 0, +\infty)$ | sudá | | $\mathbf{R}\setminus\{0\}$ | $f'(x) = \operatorname{sgn} x$ |
| | $f\colon y = [x]$ | | | $\mathbf{R}$ | $\mathbf{Z}$ | lichá | | $\mathbf{R}\setminus\{\pm k, k \in \mathbf{N}\}$ | $f'(x) = 0$ |

$f(x)^{g(x)} = \mathrm{e}^{g(x)\ln f(x)}$

LaTeX & Inkscape

# Reálné funkce jedné reálné proměnné

| | transcendentní funkce | $D(f)$ | $H(f)$ | pozn. | graf funkce $f$ | $D(f')$ | derivace $f'$ |
|---|---|---|---|---|---|---|---|
| exponenciální funkce | $f\colon y = a^x, \quad a > 0$ | $\mathbf{R}$ | $(0, +\infty)$ ($\{1\}$ pro $a = 1$) | |  | $\mathbf{R}$ | $f'(x) = a^x \ln a$ |
| | $f\colon y = \mathrm{e}^x$ | | | | | | $f'(x) = \mathrm{e}^x$ |
| | $\mathrm{e} \doteq 2{,}718281828$ | | | | | | |
| logaritmická funkce | $f\colon y = \log_a x, \quad a > 0, \ a \neq 1$ | $(0, +\infty)$ | $\mathbf{R}$ | |  | $(0, +\infty)$ | $f'(x) = \dfrac{1}{x \ln a}$ |
| | $f\colon y = \log_{\mathrm{e}} x = \ln x$ | | | | | | $f'(x) = \dfrac{1}{x}$ |
| | $f\colon y = \log_{10} x = \log x$ | | | | | | $f'(x) = \dfrac{1}{x \log 10}$ |
| goniometrické funkce | $f\colon y = \sin x$ | $\mathbf{R}$ | $\langle -1, 1 \rangle$ | period. $T = 2\pi$ lichá |  | $\mathbf{R}$ | $f'(x) = \cos x$ |
| | $f\colon y = \cos x$ | $\mathbf{R}$ | $\langle -1, 1 \rangle$ | period. $T = 2\pi$ sudá | | $\mathbf{R}$ | $f'(x) = -\sin x$ |
| | $f\colon y = \operatorname{tg} x = \dfrac{\sin x}{\cos x}$ | $\mathbf{R} \setminus \{(2k+1)\frac{\pi}{2}, k \in \mathbf{Z}\}$ | $\mathbf{R}$ | period. $T = \pi$ lichá |  | $\mathbf{R} \setminus \{(2k+1)\frac{\pi}{2}, k \in \mathbf{Z}\}$ | $f'(x) = \dfrac{1}{\cos^2 x}$ |
| | $f\colon y = \operatorname{cotg} x = \dfrac{\cos x}{\sin x}$ | $\mathbf{R} \setminus \{k\pi, k \in \mathbf{Z}\}$ | $\mathbf{R}$ | period. $T = \pi$ lichá | | $\mathbf{R} \setminus \{k\pi, k \in \mathbf{Z}\}$ | $f'(x) = \dfrac{-1}{\sin^2 x}$ |
| cyklometrické funkce | $f\colon y = \arcsin x$ | $\langle -1, 1 \rangle$ | $\langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$ | lichá |  | $(-1, 1)$ | $f'(x) = \dfrac{1}{\sqrt{1 - x^2}}$ |
| | $f\colon y = \arccos x$ | $\langle -1, 1 \rangle$ | $\langle 0, \pi \rangle$ | | | $(-1, 1)$ | $f'(x) = \dfrac{-1}{\sqrt{1 - x^2}}$ |
| | $f\colon y = \operatorname{arctg} x$ | $\mathbf{R}$ | $(-\frac{\pi}{2}, \frac{\pi}{2})$ | lichá |  | $\mathbf{R}$ | $f'(x) = \dfrac{1}{1 + x^2}$ |
| | $f\colon y = \operatorname{arccotg} x$ | $\mathbf{R}$ | $(0, \pi)$ | | | $\mathbf{R}$ | $f'(x) = \dfrac{-1}{1 + x^2}$ |
| hyperbolické funkce | $f\colon y = \sinh x = \dfrac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}$ | $\mathbf{R}$ | $\mathbf{R}$ | lichá |   | $\mathbf{R}$ | $f'(x) = \cosh x$ |
| | $f\colon y = \cosh x = \dfrac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}$ | $\mathbf{R}$ | $\langle 1, +\infty)$ | sudá | | $\mathbf{R}$ | $f'(x) = \sinh x$ |
| | $f\colon y = \operatorname{tgh} x = \dfrac{\sinh x}{\cosh x}$ | $\mathbf{R}$ | $(-1, 1)$ | lichá | | $\mathbf{R}$ | $f'(x) = \dfrac{1}{\cosh^2 x}$ |
| | $f\colon y = \operatorname{cotgh} x = \dfrac{\cosh x}{\sinh x}$ | $\mathbf{R} \setminus \{0\}$ | $\mathbf{R} \setminus \langle -1, 1 \rangle$ | lichá | | $\mathbf{R} \setminus \{0\}$ | $f'(x) = \dfrac{-1}{\sinh^2 x}$ |
| hyperbolometrické funkce | $f\colon y = \operatorname{argsinh} x$ | $\mathbf{R}$ | $\mathbf{R}$ | lichá |   | $\mathbf{R}$ | $f'(x) = \dfrac{1}{\sqrt{1 + x^2}}$ |
| | $f\colon y = \operatorname{argcosh} x$ | $\langle 1, +\infty)$ | $\langle 0, +\infty)$ | | | $(1, +\infty)$ | $f'(x) = \dfrac{1}{\sqrt{x^2 - 1}}$ |
| | $f\colon y = \operatorname{argtgh} x$ | $(-1, 1)$ | $\mathbf{R}$ | lichá | | $(-1, 1)$ | $f'(x) = \dfrac{1}{1 - x^2}$ |
| | $f\colon y = \operatorname{argcotgh} x$ | $\mathbf{R} \setminus \langle -1, 1 \rangle$ | $\mathbf{R} \setminus \{0\}$ | lichá | | $\mathbf{R} \setminus \langle -1, 1 \rangle$ | $f'(x) = \dfrac{1}{1 - x^2}$ |