Test: 12b - 50% asi na mikuláše

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$p(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n, \quad a_i \in C, \quad a_0 \neq 0$

$(x^6 - 3x^5 + 3x^4 - 6x^3 + 4x^2 - 6x + 2) : (x^4 + 2x^2 + 2) = \underline{\underline{x^2 - 3x + 1}}$

$-(x^6 \quad 2x^4 \quad 2x^2)$

$0 \ -3x^5 + x^4 - 6x^3 + 2x^2 - 6x + 2$

$-(-3x^5 \quad -6x^3 \quad -6x)$

$+x^4 + \quad 2x^2 \quad +2$

$-(x^4 + \quad 2x^2 + 2)$

0

$p(x) : q(x)$

$p(x) = q(x) \cdot s(x) + r(x)$

$s(x)$ – podíl, $r(x)$ – zbytek

$p(x) = 3x^6 + 7x^5 - 4x^3 + 4x^2 + 6x + 1$

$q(x) = x + 2$

$(3x^6 + 7x^5 - 4x^3 + 4x^2 + 6x + 1) : (x+2) = 3x^5 + x^4 - 2x^3 + 4x - 2 + \frac{5}{x+2}$

$-(3x^6 + 6x^5)$

$0 \quad x^5 - 4x^3 + 4x^2 + 6x + 1$

$-(x^5 + 2x^4)$

$0 \ -2x^4 - 4x^3 + 4x^2 + 6x + 1$

$-(2x^4 - 4x^3)$

$0 \quad 0 \quad 4x^2 + 6x + 1$

$-(4x^2 + 8x)$

$0 \ -2x + 1$

$-(-2x - 4)$

5

	3	7	0	4	4	6	1
-2	↓	-6	-2	~~4~~	~~8~~	-8	4
	3	1	0	~~10~~ 4	2		5

Hornerovo schema platí jen pro $q(x) = x - c$

$p(x) = (x-c) \cdot s(x) + r(x)$, $r(x) = r$

$p(x) = x \cdot s(x) - c \cdot s(x) + r$

$\boxed{r = p(c)}$

$p(x) = x^6 - 9x^5 + 26x^4 - 8x^3 - 97x^2 + 177x - 90$

kořeny $\in \mathbb{Z}$ - dělitelé $-90 - a_0$ $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 9, \pm 10, \pm 15, \pm 18, \pm 30, \pm 45, \pm 90$

	1	-9	26	-8	-97	177	-90
1		1	-8	+18	10	-87	90
	1	-8	18	~~10~~	-87	90	0
						24	

nezapomenout $a_0 = 2$ dopsat do výsledku!

$2x^7 + 17x^6 - 11x^5 - 153x^4 + 29x^3 + 304x^2 + 52x - 96$

	2	17	-11	-153	29	304	52	-96
1		2	19	8	-145	-115	189	241
	2	19	8	-145	-115	189	241	145
		4	42	62	-182	-306	4	112
2	2	21	31	-91	-153	2	56	16
		6	69	174	63	276	1740	5376
3	2	23	58	21	92	580	1792	

1nás 2nás 1nás
-3, -1, -8

	2	17	-11	-153	29	304	52	-96
		-6	-33	132	63	-276	-84	6
-3	2	11	-44	-21	92	28	-32	10
		4						
-2	2	-13						

	2	11	-44	-21	92
		-6	-15	177	...
-3	2	5	-59	156	...

1nás.

Výsledek $= a(x) = (x+1)^2(x-2)^2(x+3)(2x^2+15x-8)$

$x^6 - 9x^5 + 26x^4 - 8x^3 - 97x^2 + 177x - 90$

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 9, \pm 10, \pm 15, \pm 18, \pm 30, \pm 45, \pm 90$ ← možné vůči posl. číslu

(+1) $n = 1$ ... 1. nás. kořen

$p(x) = (x-1)(x^5 - 8x^4 + 18x^3 + 10x^2 - 87x + 90)$

-1: $= -1 - 8 - 18 + 10 + 87 + 90 \neq 0$ → není kořen

	1	-8	18	10	-87	90
2		2	-12	12	44	-86
	1	-6	6	22	-43	4

$\neq 0$ → není kořen

	1	-8	18	10	-87	a_0	
-2		-2	20	-76	132	-90	
	1	-10	38	-66	45	$\lfloor 0$	$\Rightarrow$ je koren

$p(x) = (x-1)(x+2)(x^4-10x^3+38x^2-66x+45)$

$\Rightarrow$ koreny musí byť 4, nebudou sudé a < 45

koreny $\pm 1, \pm 2, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45$

	1	-10	38	-66	45	
3		3	-21	51	-45	
	1	-7	17	-15	$\lfloor 0$	$\rightarrow$ je koren
		3	-12	15		
	1	-4	5	$\lfloor 0$		– násobnosť 2

$\rightarrow$ 3 nedelí 5, konec

$c_{3,4} = 3$ ... 2. nás. koren

$p(x) = (x-1)(x+2)(x-3)^2 \cdot (x^2-4x+5)$

$c_{5,6} = \frac{4 \pm \sqrt{16-20}}{2}$ ↯ $\Downarrow$

$= \frac{4 \pm i\sqrt{|-4|}}{2} = 2 \pm i$

$$\underline{\underline{p(x) = (x-1)(x+2)(x-3)^2 \cdot (x-2-i) \cdot (x-2+i)}} \text{ ... v } C$$

DCV

$p(x) = x^9 - 13x^7 - 10x^6 + 31x^5 + 52x^4 + 105x^3 + 134x^2 + 60x + 72$

$p(x) = x^4 - 16$ — binomický polynom $p(x) = x^n + a \quad a \in I$

← rozklad čtverců

$p(x) = (x^2-4)(x^2+4) = (x-2)(x+2)\ (x^2+4)$ ... v $\mathbb{R}$

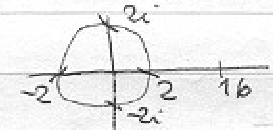
$\qquad 2 \quad -2 \quad \pm 2i$

$p(x) = (x-2)(x+2)(x-2i)(x+2i)$ .. v $\mathbb{C}$

$x^n = |z| \cdot (\cos\varphi + i\sin\varphi)$

$$\boxed{x_k = \sqrt[n]{|z|} \cdot \left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right) \quad k = 0,1,\dots,n-1}$$

$x^4 = 16$



$x^4 = 16(\cos 0 + i\sin 0)$

$x_{k+1} = \sqrt[4]{16} \cdot \left(\cos\frac{0+2k\pi}{4} + i\sin\frac{0+2k\pi}{4}\right)$

$x^6 - 1 \qquad\qquad x^6 - 7x^3 - 8$

$x^3 + i \qquad\qquad (x^3)^2 - 7x^3 - 8$ ... substituce

$x^4 + 81$

$\cancel{(x^2 - 2x + 1)(x^2 + 2x - 1)} = x^4$

$a^2 - b^2 = (a-b)(a+b)$

$p(x) = (x^4 + 2\cdot 9x^2 + 81)^2 - 18x^2 =$

$= (x^4 + 9)^2 - (3\sqrt{2}\,x)^2 =$

$= (x^2 + 9 - 3\sqrt{2}x)(x^2 + 9 + 3\sqrt{2}x) =$

$= (x^2 - 3\sqrt{2}x + 9)(x^2 + 3\sqrt{2}x + 9) =$

$x_{1,2} = \dfrac{+3\sqrt{2} \pm \sqrt{18-36}}{2} = \dfrac{3\sqrt{2} \pm i3\sqrt{2}}{2}$

$$x_{3,4} = -\frac{3\sqrt{2}}{2} \pm i\,\frac{3\sqrt{2}}{2}$$

$\boxed{\text{LAcv}}$ 11.10.

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & -3 & 4 \\ 4 & 2 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 1 \\ -2 & -1 \\ 0 & -3 \end{bmatrix}$$

Příště

$(\mathcal{L},\ +,\ \cdot\ \text{konstantou})$

$$+ : L \times L \to L$$
$$\cdot : \underset{\text{prvek}}{T} \times \underset{\text{vektor}}{L} \to L$$

$\boxed{\text{DCV}}$

P_3 ... báze
dim P_3

$q_1(x) = x^2+x+1$? LN/LZ ?
$q_2(x) = x+3$
$q_3(x) = x^2+2x-1$

18.10.

BÁZE
= členy, které generují prostor P_3 a jsou lineárně nezávislé

$ax^3 + bx^2 + cx + d \in P_3$ lib.

$$\| $$

$$a(x^3) + b(x^2) + c(x) + d(1)$$
$$= a\cdot p_1(x) + b\cdot p_2(x) + c\cdot p_3(x) + d\cdot p_4(x)$$

lin. kombinace vektorů (= součet násobků)

$p_1(x) = x^3$; $p_2(x) = x^2$; $p_3(x) = x$; $p_4(x) = 1$ = generují prostor P_3

některÿ vektor lze vyjádřit lin. kombinace zbývajících vektorů

LN/LZ ?

$$\underbrace{c_1p_1(x) + c_2p_2(x) + c_3p_3(x) + c_4p_4(x)}_{\text{lin. kombinace}} = 0$$

nulový prvek

$\forall i : c_i = 0$ (pouze)

LN

$\exists i : c_i \neq 0$

LZ

$$c_1 \cdot x^3 + c_2 \cdot x^2 + c_3 \cdot x + c_4 \cdot 1 = 0$$

$c_1 = 0$

$c_2 = 0$

$c_3 = 0$ $\Rightarrow \forall i : c_i = 0 \Rightarrow LN$

$c_4 = 0$

= tvoří bázi prostoru

$p_1(x) = x^3$; $p_2(x) = x^2$; $p_3(x) = x$; $p_4(x) = 1$

dimenze $P_3 = 4$ – počet prvků v libovolné bázi

$P_n = x^n, x^{n-1}, \ldots x^2, x, 1$ .... kanonická báze P_n

$\dim P_n = n+1$

$\mathbb{R}_n$ $[x_1, x_2, \ldots, x_n]^T \in \mathbb{R}_n$

$= x_1 [1;0;0;\ldots 0]^T$

$+ x_2 [0;1;0;\ldots 0]^T \ldots + x_n [0;0;0;\ldots 0;1]^T$

tvoří bázi $\mathbb{R}_n$

$\dim \mathbb{R}_n = n$

Řešení DCV

$q_1(x) = x^2+x+1$

$q_2(x) = x+3$

$q_3(x) = x^2+2x-1$

? LN/LZ

$c_1 p_1(x) + c_2 p_2(x) + c_3 p_3(x) = 0$

$c_1(x^2+x+1) + c_2(x+3) + c_3(x^2+2x-1) = 0$

$(c_1+c_3)x^2 + (c_1+c_2+2c_3)x + (c_1+3c_2-c_3) = 0$

$$\begin{aligned} c_1 \quad\quad + c_3 &= 0 \\ c_1 + c_2 + 2c_3 &= 0 \\ c_1 + 3c_2 - c_3 &= 0 \end{aligned}$$

$c_1 = -c_3$

$c_2 + c_3 = 0 \quad \cdot(-3)$

$3c_2 - 2c_3 = 0 \quad \}+$

$-5c_3 = 0 \Rightarrow c_3 = 0$

$c_2 = c_3 = 0$

$c_1 = -c_3 = 0$

LN

P_2

DCV

$p(x) = x^2-3x-6$ prvek P_2 $\rightarrow \widehat{p(x)}$

Jak vypadají souřadnice prvku vzhledem k dané bázi

= Co je to podprostor?

~~$c_1 x^2 \ldots c_2 3 \ldots c_3 6 = 0$~~

$x^2 + x^2 = x^2$

$x + x + 2x = -3x$

$1 + 3 - 1 = -6$

1) $a + c = 1$

2) $a + b + 2c = -3$

3) $a + 3b - c = -6$

1. $a+c=1$
2. $a+b+2c=-3$
3. $a+3b-c=-6$

$a=1-c$

$1-c+b+2c=-3$ $\qquad b=-4-c$

$1-c-4-c-c=-6$

$-3c=-6+4-1$

$c=1$

$a=0$

$1-1+b+2=-3$

$b=-5$

$\cancel{[0;-5;1]}$

$a=1-c$ $\qquad \underline{\underline{a=2}}$

$1-c+b+2c=-3$ $\qquad b=-4-c$ $\qquad b=-4+1=\underline{\underline{-3}}$

$1-c+3(-4-c)-c=-6$

$1-c-12-3c-c=-6$

$-5c=-6+12-1$

$-5c=5$

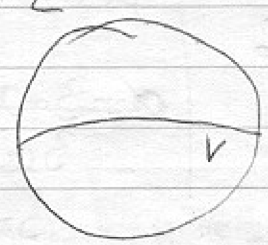
$\underline{\underline{c=-1}}$

$[2;-3;-1]$

$\hat{p}(x)=[2;-3;-1]$

25.10.

Podprostory L



~~$V\subseteq L$~~

~~$(V;+;\cdot)$ lin. pr.~~ lineární prostor

kontrola vlastností 1) $u, v \in V \Rightarrow u+v\in V$

2) $u\in V, \alpha\in T \Rightarrow \alpha\, u \in V$

Splněno 1, 2 .. V je podprostor prostoru L

$V\in P_2 = L$

$V=\{ax^2+c;\ a,c\in\mathbb{R}\}$

1) $p_1(x)=a_1x^2+c_1$

$p_2(x)=a_2x^2+c_2$

$(p_1+p_2)(x)=(a_1x^2+c_1)+(a_2x^2+c_2)=$

$\underbrace{(a_1+a_2)}_{\in\mathbb{R}}x^2+\underbrace{(c_1+c_2)}_{\in\mathbb{R}}$

2) $p(x) = ax^2 + c$; $\alpha \in \mathbb{R}$ lib.

$$\alpha \cdot p(x) = \alpha(ax^2+c) = \underbrace{\underset{\in \mathbb{R}}{(\alpha a)} x^2 + \underset{\in \mathbb{R}}{(\alpha c)}}_{\in V}$$

$\Rightarrow$ V je podprostor prostoru P_2

$W = \{ bx + 3 ; b \in \mathbb{R} \} \in P_2$

1) $p_1(x) = b_1 x + 3$ $\quad p_2(x) = b_2 x + 3$

$(p_1 + p_2)(x) = (b_1 x + 3) + (b_2 x + 3)$

$P_2 = (b_1 + b_2)x + \underline{6} \longrightarrow 3 \neq 6$! • Nepatří do W

W není podprostor

nulový polynom $\notin W$

Když je pevná konstanta zadaná, většinou nenáleží podprostor prostoru.

V ? bázov

? dim

$V = \{ ax^2 + c ; a, c \in \mathbb{R} \}$

prvky kanonické báze

$p(x) \in V$ $\quad p(x) = ax^2 + c = a x^2 + c \cdot 1$

báze V : $p_1(x) = x^2$

$p_2(x) = 1$

dim V = 2

$\boxed{\text{DCV 1}}$

$L = P_2$

podprostor V je generován prvky

$p_1(x) = x^2+2x+1$

$p_2(x) = 3x^2+x+5$

$p_3(x) = 3x^2-4x+7$

1) báze V, dim V

2) $q(x) = x^2+12x-3$? leží v podprostoru V

— pokud ano $\Rightarrow \hat{q}$

3) $r(x) = x^2+1$ ... jako u 2

$\boxed{\text{DCV 2}}$

$R_{2,2}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ 2/2 $a,b,c,d \in R$

$(R,+,\cdot)$

1 ? $A_1 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ $A_3 = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

$A_4 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ tvoří bázi ? $R_{2,2}$ = ověřit LN

2 $M = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ = ? $\hat{M}$ (souřadnice)

Standardní matice $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

(kanonické)

$\dim \mathbb{R}_{2,2} = 4$

DCV řešení

$c_1 p_1(x) + c_2 p_2(x) + c_3 p_3(x) = 0$

$c_1 + 3c_2 + 3c_3 = 0$
$2c_1 + c_2 - 4c_3 = 0$
$c_1 + 5c_2 + 7c_3 = 0$

$$\begin{bmatrix} 1 & 3 & 3 \\ 2 & 1 & -4 \\ 1 & 5 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 3 \\ 0 & -5 & -10 \\ 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

netriviální ~~nekonečné~~ řešení $\Rightarrow c_2 = -2c_3,\ c_1 = 3c_3,\ c_3 \in \mathbb{R}$ lib.

např. $c_3 = 1 \Rightarrow c_1 = 3,\ c_2 = -2$
$3p_1(x) - 2p_2(x) + p_3(x) = 0$

$\Rightarrow$ LZ

vynecháme $p_3(x) = -3p_1(x) + 2p_2(x)$

$p_1(x), p_2(x)$: $c_1 p_1(x) + c_2 p_2(x) = 0$
$(c_1 + 3c_2)x^2 + (2c_1 + c_2)x + (c_1 + 5c_2) = 0$
$c_1 + 3c_2 = 0$
$2c_1 + c_2 = 0 \Rightarrow c_1' = 0,\ c_2' = 0 \Rightarrow$ LN
$c_1 + 5c_2 = 0$

báze V: $p_1(x) = x^2 + 2x + 1,\ p_2(x) = 3x^2 + x + 5$ (např.)
dim V = 2

②

$q(x) = x^2 + 12x - 3$

$c_1 p_1(x) + c_2 p_2(x) = q(x)$
$(c_1 + 3c_2)x^2 + (2c_1 + c_2)x + (c_1 + 5c_2) = x^2 + 12x - 3$

$c_1 + 3c_2 = 1$
$2c_1 + c_2 = 12$
$c_1 + 5c_2 = -3$

$$\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 2 & 1 & 12 \\ 1 & 5 & -3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & -5 & 10 \\ 0 & 2 & -5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right]$$

$c_2 = -2;\ c_1 = 7$

$\Rightarrow q \in V \ , \ \hat{q}(x) = \begin{bmatrix} 7 \\ -2 \end{bmatrix} = [7, -2]^T$

? $r(x) = x^2 + 1$

$c_1 p_1(x) + c_2 p_2(x) = r(x)$

$\hookrightarrow$ $c_1 + 3c_2 = 1$
$2c_1 + c_2 = 0$
$c_1 + 5c_2 = 1$

$$\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 2 & 1 & 0 \\ 1 & 5 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & -5 & 2 \\ 0 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{array}\right]$$

soustava nemá řešení
$0 \cdot c_1 + 0 \cdot c_2 \neq -2$

$\Rightarrow r \notin V$

$\boxed{\text{DCU 2}}$

$c_1 A_1 + c_2 A_2 + c_3 A_3 + c_4 A_4 = 0$

$$c_1 \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + c_4 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} c_1 + c_2 + c_3 + 2c_4 & , & c_2 + c_3 + c_4 \\ 2c_1 + c_2 + 2c_3 + c_4 & , & c_1 + 2c_2 + 2c_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$c_1 + c_2 + c_3 + 2c_4 = 0$
$c_2 + c_3 + c_4 = 0$
$2c_1 + c_2 + 2c_3 + c_4 = 0$
$c_1 + 2c_2 + 2c_4 = 0$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & 1 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -2 & -1 \end{bmatrix}$$

$\Downarrow$

$c_1 = 0, c_2 = 0, c_3 = 0, c_4 = 0$ $\sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -5 \end{bmatrix}$

$\Rightarrow LN$

$\Rightarrow A_1, A_2, A_3, A_4$ jsou báze v $\mathbb{R}_{2,2}$

$M = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

to samé jen místo $= 0$ ... $= \pi$

$\Rightarrow c_1 = 1;\ c_2 = -1;\ c_3 = -1;\ c_4 = 1$

$\Rightarrow \overset{1}{M} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = [1; -1; -1; 1]^T$

× kanonická báze $\overset{1}{M} = [1; -1; 0; 1]^T$

L+CV 1.11.

$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 3 \end{bmatrix}^{\nearrow 3/2}$, $B = \begin{bmatrix} 7 & -3 & 4 \\ 4 & 2 & 0 \end{bmatrix}^{\nearrow 2/3}$, $C = \begin{bmatrix} 2 & 1 \\ -2 & -1 \\ 0 & -3 \end{bmatrix}^{\nearrow 3/2}$

$(1\cdot7)+(0\cdot4)$

$A \cdot B = \begin{bmatrix} 7 & -3 & 4 \\ 10 & -4 & 8 \\ 12 & 6 & 0 \end{bmatrix}$

$3/2 \,//\, 2/3 \Rightarrow 3/3$

Násobení řádek A · sloupec B

$B \cdot A = \begin{bmatrix} 11 & 9 \\ 8 & 2 \end{bmatrix}$

$\begin{bmatrix} 7 & -3 & 4 \\ 4 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 3 \end{bmatrix}$

$7 - 6 \quad 0 \quad -3+12$

$4 + 8$

$\boxed{A \cdot B \neq B \cdot A}$! obecně

DETERMINANTY

$\det \begin{bmatrix} 1 & 9 \\ 8 & 2 \end{bmatrix} = 1\cdot2 - 9\cdot8 = -70$

$- \quad +$

$\det A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ $\det = \underbrace{\cos^2\alpha - (-\sin^2\alpha)}^{\text{vzoreček}} = 1 \quad \forall \alpha \in \mathbb{R}$

Determinant se ~~z~~ nezmění když k řádku přičteme k-násobek pivotního řádku 8.11.

$$A \quad \begin{bmatrix} 1 & 2 & 0 & 1 \\ 3 & -2 & 1 & 0 \\ 0 & 6 & 3 & -2 \\ 2 & 4 & 3 & 1 \end{bmatrix} \begin{array}{l} \cdot 2 \quad \cdot(-1) \\ +3ř. \\ \\ \\ \end{array}$$

$$\det A = \det \begin{bmatrix} 1 & 2 & 0 & \boxed{1} \\ 3 & -2 & 1 & 0 \\ 2 & 10 & 3 & 0 \\ 1 & 2 & 3 & 0 \end{bmatrix} = 1\cdot(-1)^{1+4} \cdot \det \begin{bmatrix} 3 & -2 & 1 \\ 2 & 10 & 3 \\ 1 & 2 & 3 \end{bmatrix} +0+0+0$$

vytkneme
- pouze celý řádek/sloupec

$$= (-1)\cdot 2 \cdot \det \begin{bmatrix} 3 & -1 & 1 \\ 2 & 5 & 3 \\ 1 & 1 & 3 \end{bmatrix} = -2\cdot\det \begin{bmatrix} 3 & -4 & -8 \\ 2 & 3 & -3 \\ \boxed{1} & 0 & 0 \end{bmatrix}$$

(-1) (-3)

$$= -2 \cdot 1 \cdot (-1)^{3+1} \cdot \det \begin{bmatrix} -4 & -8 \\ 3 & -3 \end{bmatrix} = -2 \cdot (-4) \cdot 3 \cdot \det \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$$

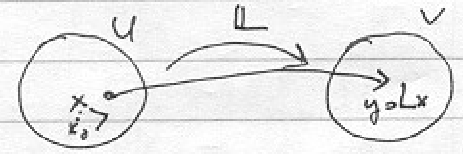
$$= 24 \cdot (-1-2) = \underline{\underline{-72}}$$

DCV

$$\det \begin{bmatrix} 2 & 1 & 4 & 3 & 5 \\ 3 & 4 & 0 & 5 & 0 \\ 3 & 4 & 5 & 2 & 1 \\ 1 & 5 & 2 & 4 & 3 \\ 4 & 6 & 0 & 7 & 0 \end{bmatrix}$$

$$\det \begin{bmatrix} 3 & 2 & 4 & 2 & 2 \\ 4 & 3 & 6 & 5 & 3 \\ -3 & -2 & -3 & -3 & -4 \\ 2 & 3 & 3 & 2 & 4 \\ -4 & -3 & -5 & -2 & -3 \end{bmatrix}$$

Lineární zobrazení



$\mathbb{L}$ lin. zobrazení U ... V

1) $\forall x_1, x_2 \in U : \mathbb{L}(x_1 + x_2) = \mathbb{L}(x_1) + \mathbb{L}(x_2)$

2) $\forall x \in U, \forall \lambda \in T : \mathbb{L}(\lambda x) = \lambda \cdot \mathbb{L}x$

$\mathbb{L} : R_{2,2} \to P_2$

$\mathbb{L}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a + 2b)x^2 + (2a - c)x + (c + 4d)$

a) ? lineární

$A_1 = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}, A_2 = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$

$\mathbb{L} = A_1 + A_2 = \mathbb{L}\left(\begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix}\right) = (a_1 + a_2 + 2(d_1 + d_2))x^2$
$+ 2(a_1 + a_2) - (c_1 + c_2)x + (c_1 + c_2) + 4(d_1 + d_2)$

$(a_1 + 2d_1)x^2 + (2a_1 \cancel{+} - c_1)x + (c_1 + 4d_1) + (a_2 + 2d_2)x^2$
$+ (2a_1 - c_2)x + (c_2 + 4d_2)$

$= \mathbb{L}(A_1) + \mathbb{L}(A_2)$

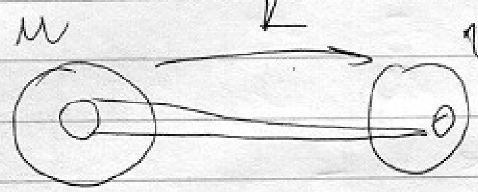
↘ obrazem součtu je součet obrazů

$\cancel{2}\ \mathbb{L}(\lambda \cdot A) = \mathbb{L}\left(\begin{bmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{bmatrix}\right) = (\lambda a + 2\lambda d)x^2 + (2\lambda a - \lambda c)x$
$+ (2\lambda c + 4\lambda d) = \lambda[(a + 2d)x^2 + (2a - c)x + (c + 4d)] = \lambda \mathbb{L}(A)$

? jádro, obraz $\mathbb{L}$

15.11.

Jádro = prvky z prostoru U, které se zobrazí do nulového prvku



kernel = jádro

$$\boxed{\ker L = \{u \in U : L(u) = 0\} \le U}$$

Podprostor

$\ker L$: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $L(A) = 0$

$$(a+2d)x^2 + (2a-c)x + (c+4d) = 0$$

$$\left.\begin{aligned} a + 2d &= 0 \rightarrow a = -2d \\ 2a - c &= 0 \\ c + 4d &= 0 \rightarrow c = -4d \end{aligned}\right\} \text{soustava 3 rovnic o 4 neznámých}$$

(2) $2(-2d) + 4d = 0$

$-4 + 4d = 0$ ✓

$A = \begin{bmatrix} -2d & b \\ -4d & d \end{bmatrix} \in \ker L$, $b, d \in \mathbb{R}$ lib

→ rozdělení $\underset{b=1}{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}$, $\underset{d=1}{\begin{bmatrix} -2 & 0 \\ -4 & 1 \end{bmatrix}}$

$\dim(\ker L) = 2$ - def L

→ 2 prvky

OBRAZ $\operatorname{Im} L$

= množina takových prvků z V které byly přiřazeny z prostoru U = OBOR HODNOT pro lin. zobrazení

Obraz

$$\boxed{\operatorname{Im} L = \{v \in V : \exists u \in U : v = L(u)\} \le V}$$

Podprostor

$$\boxed{\operatorname{Dim}(\ker L) + \operatorname{Dim}(\operatorname{Im} L) = \dim U}$$

$L(A_1) = x^2 + 2x = p_1(x)$

$L(A_2) = 0 = p_2(x)$

$L(A_3) = -x + 1 = p_3(x)$

$L(A_4) = 2x^2 + 4 = p_4(x)$

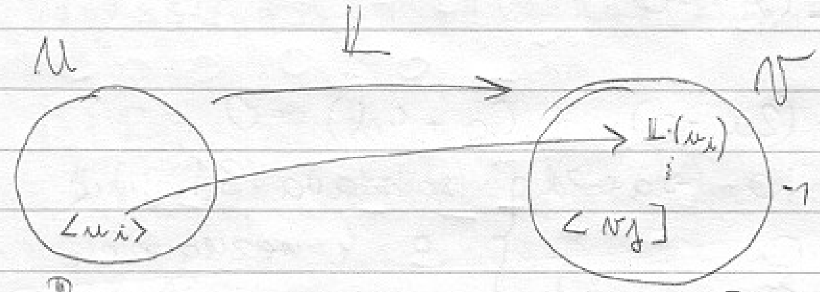
$\sin^3 x$ ~~...~~ $= \sin x$

? LN p_1, p_2, p_3 ; OCV

$p_1(x) = x^2 + 2x$

$p_3(x) = -x + 1$ ... LN ... báze $\mathrm{Im}\,\mathbb{L}$

$\mathrm{Dim}\ \mathrm{Im}\,\mathbb{L} = 2$



$\mathbb{P}_2$

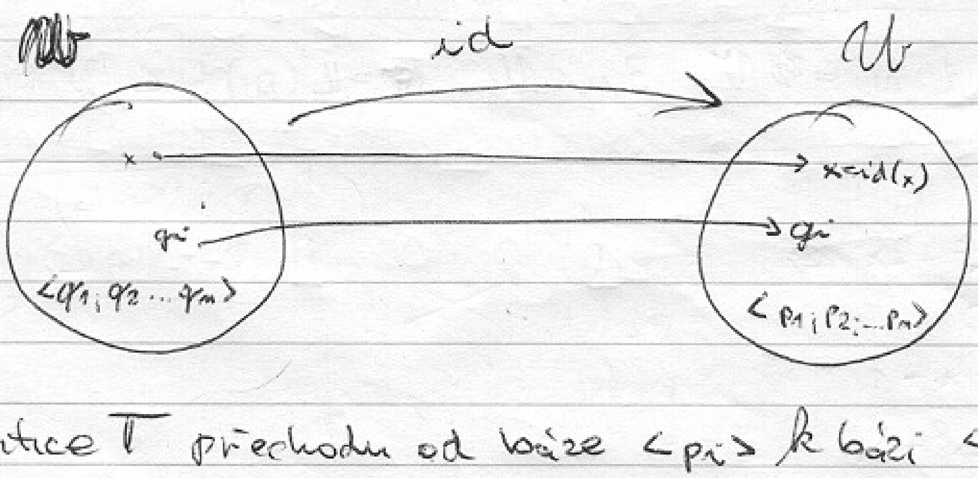
$\forall i : u_i \longrightarrow \mathbb{L}(u_i) \longrightarrow \widehat{\mathbb{L}(u_i)}$

... sloupce matice lin. zobrazení

<u>Úkol</u>: matice $\mathbb{M}'$ lin. zobrazení $\mathbb{L}$ v bázích

prostor $\mathbb{R}_{2,2}$: $\mathbb{B}_1 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $\mathbb{B}_2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$,
$\mathbb{B}_3 = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$, $\mathbb{B}_4 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

prostor $\mathbb{P}_2$: $r_1(x) = x^2 - 1$
$r_2(x) = x^2 + 1$
$r_3(x) = 2x + 3$



22.11.

matice T přechodu od báze $\langle p_i \rangle$ k bázi $\langle q_i \rangle$

= matice identického zobrazení prostoru $\underset{\langle p_i \rangle}{U}$ do $\underset{\langle q_i \rangle}{U}$

ident. zobrazení

$$q_i \longrightarrow id(q_i) = q_i \longrightarrow \widehat{q_i}_{\langle q_i \rangle}$$

$$T = \left[\, \hat{q}_1 \;\vdots\; \hat{q}_2 \;\vdots\; \dots \; \hat{q}_n \,\right]$$

$$\boxed{T\, \hat{x}_{\langle q_i \rangle} = \tilde{x}_{\langle p_i \rangle}} \qquad \boxed{T^{-1} \cdot \tilde{x} = \hat{x}}$$

$$\begin{aligned}
2x_1 - x_2 + 3x_3 + 4x_4 &= 5 \\
4x_1 - 2x_2 + 5x_3 + 6x_4 &= 7 \\
6x_1 - 3x_2 + 7x_3 + 8x_4 &= 9 \\
8x_1 - 4x_2 + 9x_3 + 10x_4 &= 11
\end{aligned}$$

$$[A|B] \quad \left[\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 4 & -2 & 5 & 6 & 7 \\ 6 & -3 & 7 & 8 & 9 \\ 8 & -4 & 9 & 10 & 11 \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & -2 & -4 & -6 \\ 0 & 0 & -3 & -6 & -9 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad hod(A) = 2 = hod([A|B])$$

3. a 4. řádek je stejný jako 2. řádek => vynulujeme

$x_1 = 0$ $\quad x_3 = 3$ $\quad x_4 = 0$ $\quad 5-4$ $\quad -4/2 = 2$

$$x = [-2, 0, 3, 0]^T + k_1 \cdot \left[\tfrac{1}{2}, 1, 0, 0\right]^T \; k_2 \, [1, 0, -2, 1]^T$$

$$Ax = b \; \dots \qquad Ax = 0$$

2 Dom. úkolu si spočíst soustavu, která má nek. mnoho řešení

28. 11.

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 2 & 6 & -11 & 3 \\ 1 & -2 & -7 & +31 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 2 & -5 & -13 \\ 0 & -4 & 10 & 26 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 2 & -5 & -13 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad H(A|b) = 2$$

dim $3-2=1$

$$x\left[18 ; \frac{-13}{2} ; \underline{0}\right]^T + k\cdot\left[-4 ; 5 ; \underline{2}\right]$$

$A\cdot x = b$

LN řešení, kdy pravá strana $=0$

$$\left[\begin{array}{ccccc|c} 4 & 8 & 8 & -6 & 4 & 9 \\ 2 & 4 & 10 & -6 & 2 & 15 \\ 1 & 2 & 3 & -2 & 1 & 4 \\ 3 & 6 & 5 & -5 & 4 & 10 \end{array}\right] \sim \begin{array}{c} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & -2 & 1 & 4 \\ 2 & 4 & 10 & -6 & 2 & 15 \\ 3 & 6 & 5 & -5 & 4 & 10 \\ 4 & 8 & 8 & -6 & 4 & 9 \end{array}\right] \end{array} \begin{array}{c} \\ -2I. \\ -3I. \\ -4I. \end{array} \sim$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 2 & 3 & -2 & 1 & 4 \\ 0 & 0 & 4 & -2 & 0 & 7 \\ 0 & 0 & -4 & 1 & 1 & -2 \\ \cancel{0} & \cancel{0} & \cancel{-4} & \cancel{2} & \cancel{0} & \cancel{-7} \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & 2 & 3 & -2 & 1 & 4 \\ 0 & 0 & 4 & -2 & 0 & 7 \\ 0 & 0 & 0 & -1 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\frac{27}{4} \cdot 2 \cdot \frac{27-4}{4}$

$H(A|b) = 3 \qquad dim = 5-3 = 2$

$$\left[\frac{-15}{4} ; 0 ; \frac{-3}{4} ; \cancel{-5} ; \underline{0}\right]^T$$

$$x = \left[\cancel{2} ; \underline{0} ; \cancel{2} ; \cancel{-9} ; \underline{0}\right]^T + k_1\left[\frac{26}{4} ; \underline{0} ; \frac{9}{4} ; +1 ; \underline{1}\right]^T$$

$$+ k_2\left[-2 ; \underline{1} ; 0 ; 0 ; \underline{0}\right]^T$$

asi :o)

$$\begin{bmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \cdot X = \begin{bmatrix} 10 & 9 & 2 \\ 0 & -3 & 3 \\ 3 & 7 & -4 \end{bmatrix}$$

$$A \cdot A^{-1} = A^{-1} \cdot A = I$$ ← jednotková matice

$A \cdot \cancel{X} X = B \quad / \cdot A^{-1}$ zleva!

$$\underbrace{(A^{-1} \cdot A)}_{I} \cdot X = A^{-1} \cdot B$$

$$X = A^{-1} \cdot B$$

1) $[A | I] \sim [I / A^{-1}]$ Gauss-Jordanova eliminace

2) determinant pomocí adjungované matice ! $n = 2, 3$!

$$A^{-1} = \frac{1}{\det A} \cdot \operatorname{adj} A$$

← transponovaná matice k matici algebraických doplňků

$$\det A = -1, \quad \operatorname{adj} A = \begin{bmatrix} -1 & -1 & 1 \\ 4 & 5 & -6 \\ 3 & 3 & -4 \end{bmatrix}^T = \begin{bmatrix} -1 & 4 & 3 \\ -1 & 5 & 3 \\ 1 & -6 & -4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-1} \cdot \begin{bmatrix} -1 & 4 & 3 \\ -1 & 5 & 3 \\ 1 & -6 & -4 \end{bmatrix} = \begin{bmatrix} 1 & -4 & -3 \\ 1 & -5 & -3 \\ -1 & 6 & 4 \end{bmatrix}$$

Dev – násobení matic dodělat

$$Y \cdot A = B \qquad Y \cdot A \cdot A^{-1} = B \cdot A^{-1}$$

Řešení domácích

úloh (snad
správné :o))

STEWER

$p(x) = x^9 - 13x^7 - 10x^6 + 31x^5 + 52x^4 + 105x^3 + 134x^2 + 60x + 72$

kořeny : $\underset{\times}{\pm 1}$, ± 2, ± 3, ± 4, ± 6, ± 8, ± 9, ± 12, ± 18, ± 24, ± 36, ± 72

	1	0	-13	-10	31	52	105	134	60	72	
1		1	1	-12	-22	9	61	166	300	360	
	1	1	-12	-22	9	61	166	300	360	432	$\neq 0$

	1	0	-13	-10	31	52	105	134	60	72	
-1		-1	1	12	-2	-29	-23	-82	-52	-8	
	1	-1	-12	2	29	23	82	52	8	64	$\neq 0$

	1	0	-13	-10	31	52	105	134	60	72	
2		2	4	-18	-56	-50	4	218	704	1528	
	1	2	-9	-28	-25	2	109	352	764	1600	$\neq 0$

	1	0	-13	-10	31	52	105	134	60	72	
-2		-2	4	+18	-16	-30	-44	-122	-24	-72	
	1	-2	-9	8	15	22	61	12	36	0	$= 0$
-2		-2	8	2	-20	10	-64	6	-36		
	1	-4	-1	10	-5	32	-3	18	0	$= 0$	
-2		-2	12	-22	+24	-38	12	-18			
	1	-6	11	-12	+19	-6	9	0	$= 0$		

$p(x) = (x+2)^3(x^6 - 6x^5 + 11x^4 - 12x^3 + 19x^2 - 6x + 9)$

kořeny < 9 $\quad c: \pm 3, \pm 9$

	1	-6	11	-12	19	-6	9	
3		3	-9	6	-18	3	-9	
	1	-3	2	-6	1	-3	$\boxed{0}$	$=0$
3		3	0	6	0	3		
	1	0	2	0	1	$\boxed{0}$	$=0$	

$p(x) = (x+2)^3 \cdot (x-3)^2 \, (x^4+2x^2+1) \; \ldots v \; \mathbb{R} = \underline{(x+2)^3(x-3)^2(x^2+1)^2}$

$\rightarrow \; t = x^2$ $\qquad -1 = x^2$

$x = \sqrt{-1}$

$\underline{x = \pm i}$

$t^2 + 2t + 1 = 0$

$D = 4 - 4 = 0$

$t_{1,2} = \dfrac{-2 \pm 0}{2} = -1$

$(x+2)^3(x-3)^2(x-i)^2(x+i)^2 \; \ldots v \; \mathbb{C}$

LaLV

Steiner

$\boxed{\text{Dcv 2}}$

$R_{2,2}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ 2/2 $a,b,c,d \in R$

$A_1 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ $A_3 = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ $A_4 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$a_1(x) = x^3 + 2x + 1$

$a_2(x) = x^3 + x^2 + x + 2$

$a_3(x) = x^3 + x^2 + 2x$

$a_4(x) = 2x^3 + x^2 + x + 2$

$c_1 a_1(x) + c_2 a_2(x) + c_3 a_3(x) + c_4 a_4(x) = 0$

$c_1(x^3+2x+1) + c_2(x^3+x^2+x+2) + c_3(x^3+x^2+2x) +$

$+ c_4(2x^3+x^2+x+2) = 0$

$x^3(c_1 + c_2 + c_3 + 2c_4)$

$x^2(c_2 + c_3 + c_4)$

$x(2c_1 + c_2 + 2c_3 + c_4)$

$c_1 + 2c_2 + 2c_4$

$c_1 + c_2 + c_3 + 2c_4 = 0$

$c_2 + c_3 + c_4 = 0$ $\Leftarrow$

$2c_1 + c_2 + 2c_3 + c_4 = 0$

$c_1 + 2c_2 \quad + 2c_4 = 0$

② $\rightarrow c_2 = -c_3 - c_4 \Rightarrow c_2 = 0$

① $c_1 \cancel{-c_3} - c_4 \cancel{+c_3} + 2c_4 = 0$

$c_1 + c_4 = 0 \rightarrow \underline{c_1 = -c_4} \Rightarrow c_1 = 0$

③ $2(-c_4) - c_3 \cancel{-c_4} + 2c_3 \cancel{+c_4} = 0$

$-2c_4 = -c_3 \rightarrow \underline{c_3 = 2c_4} \Rightarrow c_3 = 0$

④ $-c_4 + 2(-2c_4 - c_4) + 2c_4 = 0$

$-c_4 - 4c_4 - \cancel{2c_4} + \cancel{2c_4}$ $\quad -5c_4 = 0 \rightarrow \underline{\underline{c_4 = 0}}$

$\left.\begin{array}{l} c_1 = 0 \\ c_2 = 0 \\ c_2 = 0 \\ c_3 = 0 \\ c_4 = 0 \end{array}\right\}$ LN

~~tvoří bázi~~

tvoří bázi

Není celé ...

②

$M = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$m(x) = x^3 - x^2 + 1$

$c_1 + c_2 + c_3 + 2c_4 = 1$

$c_2 + c_3 + c_4 = -1$

$2c_1 + c_2 + 2c_3 + c_4 = 0$

$c_1 + 2c_2 + 2c_4 = 1$

② $c_2 = -1 - c_3 - c_4 \Rightarrow c_2 = -1 - 1 + 2c_1 - c_4$

$c_2 = -2 + 2c_1 - c_4$

$\downarrow$

$c_2 = -2 + 2c_1 - (2 - c_1)$

$= -2 + 2c_1 - 2 + c_1$

$c_2 = 3c_1 - 4 \Rightarrow 3 - 4 \Rightarrow c_2 = -1$

③ $2c_1 - 1 - c_3 - \cancel{c_4} + 2c_3 + \cancel{c_4} = 0$

$2c_1 + c_3 = 1 \quad c_3 = 1 - 2c_1 \Rightarrow 1 - 2 \Rightarrow c_3 = -1$

① $c_1 - 2 + \cancel{2c_1} - c_4 \cancel{+1} - \cancel{2c_1} + 2c_4 = \cancel{1}$

$c_1 + c_4 = 2$

$c_4 = 2 - c_1 \Rightarrow c_4 = 1$

$\hat{M} = [1; -1; -1; 1]^T$

④ $c_1 + 2(3c_1 - 4) + 2(2 - c_1) = 1$

$c_1 + 6c_1 - 8 + 4 - 2c_1 = 1$

$5c_1 = 1 + 8 - 4$

$5c_1 = 5$

$c_1 = 1$

DCV

Steiner

$$\begin{vmatrix} 2 & 1 & 4 & 3 & 5 \\ 3 & 4 & 0 & 5 & 0 \\ 3 & 4 & 5 & 2 & 1 \\ 1 & 5 & 2 & 4 & 3 \\ 4 & 6 & 0 & 7 & 0 \end{vmatrix} \begin{matrix} -5 \\ \\ \\ -3 \\ \\ \end{matrix} \approx \begin{vmatrix} -13 & -19 & -21 & -7 & 0 \\ 3 & 4 & 0 & 5 & 0 \\ 3 & 4 & 5 & 2 & 1 \\ -8 & -7 & -13 & -2 & 0 \\ 4 & 6 & 0 & 7 & 0 \end{vmatrix}$$

$$\approx 1 \cdot \cancel{(-1)}^{3+5} \begin{vmatrix} -13 & -19 & -21 & -7 \\ 3 & 4 & 0 & 5 \\ -8 & -7 & -13 & -2 \\ 4 & 6 & 0 & 7 \end{vmatrix} \begin{matrix} -1 \\ \\ \\ \\ \end{matrix} \approx \begin{vmatrix} -5 & -12 & -8 & -5 \\ 3 & 4 & 0 & 5 \\ -8 & -7 & -13 & -2 \\ 4 & 6 & 0 & 7 \end{vmatrix}$$

$$\approx (-13) \cdot \cancel{(-1)}^{3+3} \begin{vmatrix} -5 & -12 & -5 \\ 3 & 4 & 5 \\ 4 & 6 & 7 \end{vmatrix} \begin{matrix} +1 \\ \\ \\ \end{matrix} + (-8)\cancel{(-1)}^{1+3} \begin{vmatrix} 3 & 4 & 5 \\ -8 & -7 & -2 \\ 4 & 6 & 7 \end{vmatrix} \begin{matrix} \\ \\ -1 \end{matrix}$$

$$\approx (-13) \begin{vmatrix} -2 & -8 & 0 \\ 3 & 4 & 5 \\ 4 & 6 & 7 \end{vmatrix} \begin{matrix} :(-2) \\ \\ \\ \end{matrix} + (-8) \begin{vmatrix} 3 & 4 & 5 \\ -8 & -7 & -2 \\ 1 & 2 & 2 \end{vmatrix}$$

$$\approx (-13)(-2) \begin{vmatrix} 1 & 4 & 0 \\ 3 & 4 & 5 \\ 4 & 6 & 7 \end{vmatrix} \begin{matrix} -4 \\ \\ \\ \end{matrix} + (-8) \begin{vmatrix} 3 & 4 & 5 \\ -8 & -7 & -2 \\ 1 & 2 & 2 \end{vmatrix} \begin{matrix} \\ \\ -2 \quad -2 \end{matrix}$$

$$\approx 26 \cdot \begin{vmatrix} 1 & 0 & 0 \\ 3 & -8 & 5 \\ 4 & -10 & 7 \end{vmatrix} + (-8) \begin{vmatrix} 3 & -2 & -1 \\ -8 & 9 & 14 \\ 1 & 0 & 0 \end{vmatrix}$$

$$\approx 26 \cdot 1 \cdot \cancel{(-1)}^{1+1} \begin{vmatrix} -8 & 5 \\ -10 & 7 \end{vmatrix} + (-8) \cdot 1 \cancel{(-1)}^{3+1} \begin{vmatrix} -2 & -1 \\ 9 & 14 \end{vmatrix}$$

$$= 26 \cdot (-56 + 50) + (-8)(-28 + 9)$$

$$= 26 \cdot (-6) + (-8)(-19)$$

$$= -156 + 152 = \underline{\underline{-4}}$$

$$\begin{vmatrix} 3 & 2 & 4 & 2 & 2 \\ 4 & 3 & 6 & 5 & 3 \\ -3 & -2 & -3 & -3 & -4 \\ 2 & 3 & 3 & 2 & 4 \\ -4 & -3 & -5 & -2 & -3 \end{vmatrix} \begin{array}{l} \\ \cdot 2 \\ \cdot(-3) \\ \cdot 3 \\ \cdot(-2) \end{array} \sim -\frac{1}{36} \begin{vmatrix} 3 & 2 & 4 & 2 & 2 \\ 8 & 6 & 12 & 10 & 6 \\ 9 & 6 & 9 & 9 & 12 \\ 6 & 9 & 9 & 6 & 12 \\ 8 & 6 & 10 & 4 & 6 \end{vmatrix} \begin{array}{l} \\ -3 \\ -6 \\ -6 \\ -3 \end{array}$$

$$\sim -\frac{1}{36} \cdot \begin{vmatrix} \cancel{3} & \cancel{2} & \cancel{4} & \cancel{2} & \cancel{2} \\ -1 & 0 & 0 & 4 & 0 \\ -9 & -6 & -15 & -3 & 0 \\ -12 & -3 & -15 & -6 & 0 \\ -1 & 0 & -2 & -2 & 0 \end{vmatrix} \sim \frac{-1}{36} \cdot 2 \cdot \cancel{(-1)^{1+5}} \cdot \begin{vmatrix} -1 & 0 & 0 & 4 \\ -9 & -6 & -15 & -3 \\ -12 & -3 & -15 & -6 \\ -1 & 0 & -2 & -2 \end{vmatrix}$$

$$\sim \begin{vmatrix} 1 & 0 & 0 & 0 \\ -9 & -6 & -15 & -39 \\ -12 & -3 & -15 & -54 \\ -1 & 0 & -2 & -6 \end{vmatrix} \sim \frac{-1}{36} \cdot 2 \cdot 1 \cancel{(-1)^{1+1}} \cdot \begin{vmatrix} -6 & -15 & -39 \\ -3 & -15 & -54 \\ 0 & -2 & -6 \end{vmatrix}$$
$$\begin{array}{ccc} :(-3) & & :(-3) \end{array}$$

$$\sim -\frac{2}{36} \cdot (+9) \cdot \begin{vmatrix} 2 & -15 & 13 \\ 1 & -15 & 18 \\ 0 & -2 & 2 \end{vmatrix} :2 \sim -\frac{18}{36} \cdot 2 \cdot \begin{vmatrix} 2 & -15 & 13 \\ 1 & -15 & 18 \\ 0 & -1 & 1 \end{vmatrix}$$
$$1+$$

$$\sim -\frac{36}{36} \cdot \begin{vmatrix} 2 & -2 & 13 \\ 1 & 3 & 18 \\ 0 & 0 & 1 \end{vmatrix} \sim -1 \cdot 1 \cancel{(-1)^{3+3}} \begin{vmatrix} 2 & -2 \\ 1 & 3 \end{vmatrix}$$

$$= -1 \cdot (6+2) = \underline{\underline{-8}}$$

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prostor $\mathbb{R}_{2,2}$: $\mathbb{B}_1 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $\mathbb{B}_2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$,

$\mathbb{B}_3 = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$, $\mathbb{B}_4 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

prostor P_2 :

$r_1(x) = x^2 - 1$

$r_2(x) = x^2 + 1$

$r_3(x) = 2x + 3$

$\mathbb{L}(\mathbb{B}_1) = 3x^2 + 6$

predpis : $(a+2d)x^2 + (2a-c)x + (c+4d)$

$c_1 r_1(x) + c_2 r_2(x) + c_3 r_3(x) = \mathbb{L}(\mathbb{B}_1)$

$c_1(x^2-1) + c_2(x^2+1) + c_3(2x+3) = 3x^2 + 6$

$x^2(c_1 + c_2) + 2c_3 x + (-c_1 + c_2 + 3c_3) = 3x^2 + 6$

$$\begin{aligned} c_1 + c_2 \quad &= 3 \\ 2c_3 &= 0 \\ -c_1 + c_2 + 3c_3 &= 6 \end{aligned} \qquad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 0 & 2 & 0 \\ -1 & 1 & 3 & 6 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 0 & 2 & 0 \\ -1 & 1 & 3 & 6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 3 & 9 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 2 & 3 & 9 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\Rightarrow$ $c_3 = 0$, $c_2 = \frac{9}{2}$, $c_1 = -\frac{3}{2}$

$\widehat{\mathbb{L}(\mathbb{B}_1)} = \begin{bmatrix} -\frac{3}{2} \\ \frac{9}{2} \\ 0 \end{bmatrix}$

$\mathbb{L}(\mathbb{B}_2) = 5x^2 + x + 9$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 0 & 2 & 0 \\ -1 & 1 & 3 & 9 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 3 & 14 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 2 & 3 & 14 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$c_3 = 0$ $\qquad$ $2c_2 = 14$ $\qquad$ $c_1 + 7 = 5$

$c_2 = 7$ $\qquad$ $c_1 = -2$

$\mathbb{L}(\mathbb{B}_2) = \begin{bmatrix} -2 \\ 7 \\ 0 \end{bmatrix}$

$\mathbb{L}(\mathbb{B}_3) = x^2 + 2$

$\mathbb{L}(\mathbb{B}_3)$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ -1 & 1 & 3 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 3 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 2 & 3 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$c_3 = 0 \qquad 2c_2 = 3 \qquad c_1 + \frac{3}{2} = 1$

$c_2 = \frac{3}{2} \qquad c_1 = 1 - \frac{3}{2}$

$c_1 = -\frac{1}{2}$

$\widehat{\mathbb{L}(\mathbb{B}_3)} = \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \\ 0 \end{bmatrix}$

$\mathbb{L}(\mathbb{B}_2) =$ $\qquad \mathbb{L}(\mathbb{B}_4) = 6x^2 + 3x + 9$

$= \cancel{2x^2 + x + 3}$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 6 \\ 0 & 0 & 2 & 13 \\ -1 & 1 & 3 & 39 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 6 \\ 0 & 0 & 2 & 3 \\ 0 & 2 & 3 & 15 \end{array}\right] \sim$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 2 & 3 & 15 \\ 0 & 0 & 2 & 1 \end{array}\right]$

$\cancel{2+1+3} = 5$

$6+3+9$

$2c_3 = 1$

$c_3 = \frac{3}{2}$

$\widehat{\mathbb{L}(\mathbb{B}_4)} = \begin{bmatrix} -1 \\ 3 \\ \frac{1}{2} \end{bmatrix} \qquad 2c_2 + \frac{9}{2} = 15$

$2c_2 = 15 - \frac{30}{2} \qquad 2c_2 = \frac{7}{2}$

$c_1 = 2 - 3 \qquad 2c_2 = \frac{30-9}{2} \qquad c_2 = \frac{7}{2} \cdot \frac{1}{2}$

$c_1 = -1 \qquad 2c_2 = \frac{21}{2} \qquad c_2 = 3 \quad \frac{7}{4}$

$c_2 = \frac{21}{4}$

$M' = \begin{bmatrix} -\frac{3}{2} & -2 & -\frac{1}{2} \\ \frac{9}{2} & 7 & \frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$

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$$\left(\begin{array}{ccccc|c} 1 & -1 & 2 & 3 & 4 & 0 \\ 2 & -2 & 3 & 8 & 8 & 0 \\ -1 & 1 & 0 & -7 & 1 & 0 \\ 3 & -3 & 2 & 17 & 2 & 0 \end{array}\right)\begin{array}{l} \\ -2\mathrm{I} \\ +\mathrm{I} \\ -3\mathrm{I} \end{array} \simeq \begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 2 & -4 & 5 \\ 0 & 0 & -4 & 8 & -10 \end{pmatrix} \simeq \begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 2 & -4 & 5 \\ \cancel{0} & \cancel{0} & \cancel{-2} & \cancel{4} & \cancel{-5} \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 2 & -4 & 5 \end{pmatrix}\begin{array}{l} \\ \\ 2\mathrm{II} \end{array} \simeq \begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

$H(A) = 3$

$\dim A = 2$

1.
$x_5 = 0$
$x_4 = 0$
$x_3 = 0$
$x_2 = 1$
$x_1 = 1$

2.
$x_5 = 0$
$x_4 = 1$
$x_3 = 2$
$x_2 = 0$
$x_1 = -7$

$$X = k_1[1;1;0;0;0]^T + k_2[-7;0;2;1;0]^T$$

$$\left(\begin{array}{ccccc|c} 1 & 2 & -1 & 3 & 4 & 1 \\ 2 & 4 & -3 & 5 & 13 & -3 \\ -3 & -6 & 2 & -9 & -3 & -10 \\ -1 & -2 & 3 & 4 & 6 & -1 \\ 1 & 2 & 0 & 6 & 7 & 2 \\ 4 & 8 & -5 & 8 & 9 & 5 \end{array}\right)\begin{array}{l} \\ -2\mathrm{I} \\ 3\mathrm{I} \\ +\mathrm{I} \\ -1\mathrm{I} \\ -4\mathrm{I} \end{array} \simeq \left(\begin{array}{ccccc|c} 1 & 2 & -1 & 3 & 4 & 1 \\ 0 & 0 & -1 & -1 & 5 & -5 \\ 0 & 0 & -1 & 0 & 9 & -7 \\ 0 & 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & -1 & -4 & -7 & 1 \end{array}\right)\begin{array}{l} \\ -\mathrm{III} \\ \\ 2\mathrm{III} \\ +\mathrm{III} \\ -\mathrm{III} \end{array}$$

$$\simeq \left(\begin{array}{ccccc|c} 1 & 2 & -1 & 3 & 4 & 1 \\ 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & -1 & 0 & 9 & -7 \\ 0 & 0 & 0 & 1 & 20 & -14 \\ 0 & 0 & 0 & 3 & 12 & -6 \\ 0 & 0 & 0 & -4 & -16 & 8 \end{array}\right) \simeq \left(\begin{array}{ccccc|c} 1 & 2 & -1 & 3 & 4 & 1 \\ 0 & 0 & -1 & 0 & 9 & 7 \\ 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 1 & 20 & -14 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{1} & \cancel{4} & \cancel{-2} \end{array}\right)\begin{array}{l} \\ \\ \\ -\mathrm{III} \\ \\ \end{array} \simeq$$

$$\left(\begin{array}{ccccc|c} 1 & 2 & -1 & 3 & 4 & 1 \\ 0 & 0 & -1 & 0 & 9 & 7 \\ 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 16_4 & -12_3 \end{array}\right)$$

$H(A) = 4$

$D(A) = 1$

$$x = [14;0;7;-2;0]^T + k_1[-2;1;0;0;0]^T + k_2[17;0;9;-4;1]^T$$

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VÁCLAV STEINER
30.11.2006 13:53