$\boxed{2\heartsuit}$ 1)

$a \oplus b = a \oplus b + ab$

a) ~~asociativita~~ asociativita $(a \circ b) \circ c = a \circ (b \circ c)$

$(a+b+ab) \oplus c = a+b+ab+c+ac+bc+abc$

$a \oplus (b+c+cb) = a+b+c+cb+ab+ac+abc$

$\Rightarrow$ SPLNĚNO

b) neutrální prvek $a \circ e = a$

~~a+e+ae=a~~

$a+e+ae=a$

$e \cdot (1+a) = 0$

$e = 0$

c) inverzní prvek $a \cdot a^{-1} = e$

$a + a^{-1} + a \cdot a^{-1} = e$

$a^{-1}(1+a) = e - a$

$a^{-1} = \frac{e-a}{(1+a)} \Rightarrow e = 0$

$a^{-1} = \frac{-a}{1+a} \quad a \neq -1$

$\boxed{M = \mathbb{R} \setminus \{-1\}}$

$\boxed{3 \heartsuit}$ 1)

$$\overline{\bar{A} \vee (B \wedge C)} \Rightarrow (A \Leftrightarrow (B \vee C))$$

A	B	C	$\bar{A}$	$B \wedge C$	$\bar{A} \vee (B \wedge C)$	$\overline{\bar{A} \vee (B \wedge C)}$	$B \vee C$	$A \Leftrightarrow (B \vee C)$	$\overline{\bar{A} \vee (B \wedge C)} \Rightarrow ($
1	1	1	0	1	1	0	1	1	1
1	1	0	0	0	0	1	1	1	1
1	0	1	0	0	0	1	1	1	1
1	0	0	0	0	0	1	0	0	0
0	1	1	1	1	1	0	1	0	1
0	1	0	1	0	1	0	1	0	1
0	0	1	1	0	1	0	1	0	1
0	0	0	1	0	1	0	0	1	1

Úplný disjunktní tvar

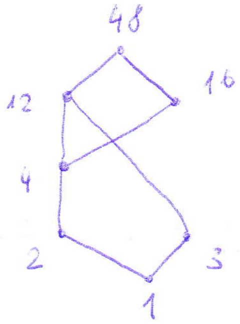
$$ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$$

Úplný konjunktní tvar

$$\bar{A} + B + C$$

$\boxed{6\heartsuit}$ 1)

$M_1 = \{1, 2, 3, 4, 12, 16, 48\}$

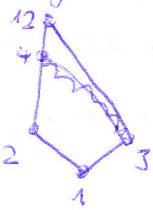


min = 1
max = 48
inf = 1
sup = 48

pro ~~všechny~~ každé 2 prvky
$\exists$ sup a inf => JE TO SVAZ

distributivní ?

obsahuje zakázaný podsvaz



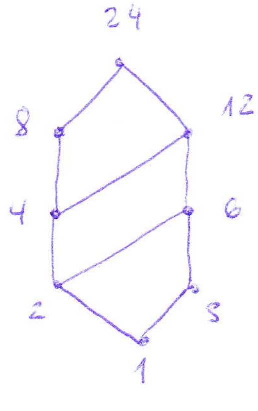
NE

komplementární ?

$\overline{1} = 48$
$\overline{48} = 1$
$\overline{2} = \nexists$

NE

$M_2 = \{1, 2, 3, 4, 6, 8, 12, 24\}$



min = 1
max = 24
inf = 1
sup = 24

pro každé 2 prvky $\exists$ sup a inf

distributivní ?

neobsahuje zakázaný podsva
M5 nebo N5

=> ANO

komplementární ?

$\overline{1} = 24$
$\overline{24} = 1$
$\overline{2} = \nexists$ => NE

$\boxed{7♡}$ 1)

$a + b\sqrt{2} \qquad a, b \in \mathbb{Q}$

a) asociativní?

$$L = [(a+b\sqrt{2})\cdot(c+d\sqrt{2})]\cdot(e+f\sqrt{2}) =$$
$$= [ac + ad\sqrt{2} + bc\sqrt{2} + bd\,2]\cdot(e+f\sqrt{2})$$
$$= ace + ade\sqrt{2} + bce\sqrt{2} + 2bde + acf\sqrt{2} + 2adf + 2bcf + 2bdf\sqrt{2}$$

$$P = (a+b\sqrt{2})\cdot[(c+d\sqrt{2})\cdot(e+f\sqrt{2})] =$$
$$= (a+b\sqrt{2})\cdot[ce + cf\sqrt{2} + de\sqrt{2} + 2df]$$
$$= ace + acf\sqrt{2} + ade\sqrt{2} + 2adf + bce\sqrt{2} + 2bcf + 2bde + 2defb\sqrt{2}$$

ANO

b) neutrální prvek

$$(a+b\sqrt{2})\cdot(e) = a+b\sqrt{2}$$
$$e = 1$$

c) inverzní prvek

$$(a+b\sqrt{2})\cdot a^{-1} = e = 1$$
$$a^{-1} = \frac{1}{a+b\sqrt{2}} \longrightarrow a+b\sqrt{2} \neq 0$$
$$a \neq -b\sqrt{2}$$
$$\frac{a}{b} \neq -\sqrt{2}$$

existuje inverzní prve

JE TO GRUPA

$\boxed{9\heartsuit}$ 2)

$$\begin{pmatrix} 1 & 3 & 2 & 1 \\ 2 & 2 & 4 & 1 \\ 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 1 \\ 4 & 2 & 4 & 3 \end{pmatrix}_{\text{mod } 5} \begin{array}{l} \\ +3\mathrm{I} \\ +2\mathrm{I} \\ +4\mathrm{I} \\ +\mathrm{I} \end{array} \sim \begin{pmatrix} 1 & 3 & 2 & 1 \\ \cancel{0} & \cancel{1} & \cancel{0} & \cancel{4} \\ 0 & 1 & 0 & 4 \\ 0 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 \end{pmatrix}_{\text{mod } 5} \sim$$

$$\sim \begin{pmatrix} 1 & 3 & 2 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 \end{pmatrix}_{\text{mod } 5} \begin{array}{l} \\ \\ \\ +\mathrm{II} \\ \\ \end{array} \sim \begin{pmatrix} 1 & 3 & 2 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ \cancel{0} & \cancel{0} & \cancel{1} & \cancel{4} \end{pmatrix}_{\text{mod } 5} \sim \begin{pmatrix} 1 & 3 & 2 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{\text{mod } 5}$$

Pouze 3 jsou LN

$\Downarrow$

NEGERUJÍ PROSTOR

(musely by být 4)

$\boxed{7\diamondsuit}$ 2)

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 2 & 0 \\ 2 & 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 1 & 1 & 1 \end{array}\right)_{\text{mod } 3} \begin{array}{l} \\ +\mathrm{I} \\ +2\mathrm{I} \\ +\mathrm{I} \end{array} \sim \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & 2 & 2 & 0 & 1 \end{array}\right)_{\text{mod } 3} \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & 2 & 2 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 & 2 \end{array}\right)_{\text{mod } 3} \begin{array}{l} \\ \\ +\mathrm{II} \\ \\ \end{array} \sim \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 & 2 \\ \cancel{0} & \cancel{0} & \cancel{2} & \cancel{0} & \cancel{0} & 2 \end{array}\right)_{\text{mod } 3}$$

$h(A) = h(A|b)$ ~~sou~~ má řešení

$$x_1 + x_2 + x_3 + x_4 + 2x_5 = 0$$
$$x_2 + x_4 = 1$$
$$2x_3 = 2$$

zvolím x_2 a x_5

$x_2 = 0$ II

$x_4 = 1$

$x_3 = 1$

$x_5 = 1$ II

$$x = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} + k \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

$x_1 + 0 + 1 + 1 + 2 = 0$

$x_1 = -4$, $x_1 = 2$

$\boxed{Q\heartsuit}$ 2)

$$\begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\begin{pmatrix} 1 & 3 & 1 & 4 & 1 \\ 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 3 & 1 & 1 \\ 4 & 1 & 0 & 2 & 2 \\ 0 & 3 & 2 & 1 & 1 \\ 1 & 2 & 0 & 3 & 1 \end{pmatrix}^{\text{mod } 5}\begin{array}{l} \\ +3\mathrm{I} \\ +3\mathrm{I} \\ +\mathrm{I} \\ \\ +4\mathrm{I} \end{array} \sim \begin{pmatrix} 1 & 3 & 1 & 4 & 1 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 4 & 1 & 1 & 3 \\ 0 & 3 & 2 & 1 & 1 \\ 0 & 4 & 4 & 4 & 0 \end{pmatrix}^{\text{mod } 5}\begin{array}{l} \\ \\ \\ \\ \\ 1:4 \end{array} \sim$$

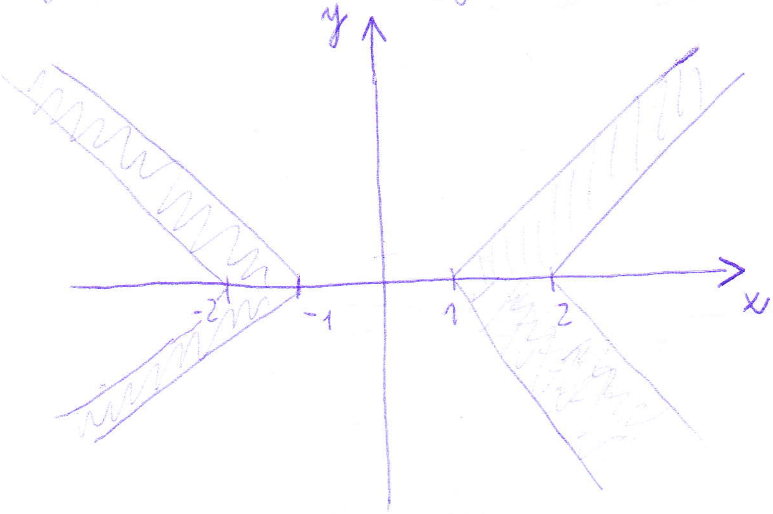
$$\sim \begin{pmatrix} 1 & 3 & 1 & 4 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 4 & 1 & 1 & 3 \\ 0 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 2 & 4 \end{pmatrix}^{\text{mod } 5}\begin{array}{l} \\ \\ \\ +\mathrm{II} \\ +2\mathrm{II} \\ \\ \end{array} \sim \begin{pmatrix} 1 & 3 & 1 & 4 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 2 & 2 & 3 \\ 0 & 0 & 4 & 3 & 1 \\ 0 & 0 & 0 & 2 & 4 \end{pmatrix}^{\text{mod } 5}\begin{array}{l} \\ \\ \\ +3\mathrm{III} \\ +\mathrm{III} \\ 1:2 \end{array} \sim$$

$$\sim \begin{pmatrix} 1 & 3 & 1 & 4 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}^{\text{mod } 5} \sim \begin{pmatrix} 1 & 3 & 1 & 4 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{\text{mod } 5}\begin{array}{l} \\ \\ \\ \\ +4\mathrm{IV} \\ \\ \end{array} \sim$$

$$\sim \begin{pmatrix} 1 & 3 & 1 & 4 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \underline{h(A)^{\text{mod } 5} = 5}$$

$\boxed{2\Diamond}$ 1)

$(x, y) \in \mathbb{R}\times\mathbb{R}\ ;\ 1 \le |x| - |y| \le 2$



$\frac{1}{2}$

$1 - 0{,}5$

$1 - \frac{1}{2}$

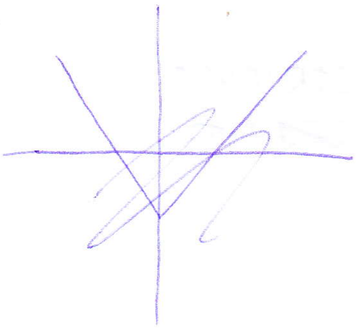
$\frac{2-1}{2}\quad \frac{1}{2}$

$x = 2$

$y = \frac{1}{2}$

$2 - \frac{1}{2}$

$\frac{4-1}{2}\quad \frac{3}{2}$

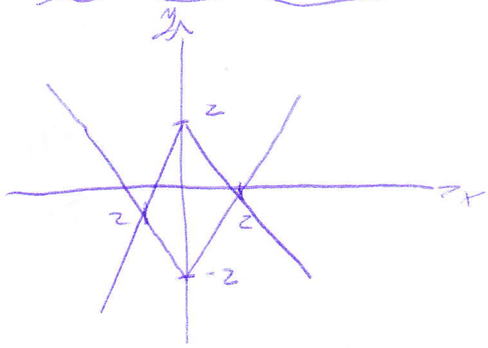


$|x| - |y| \le 2$

$-|y| \le 2 - |x|$

$|y| \ge -2 + |x|$

$y \ge \pm(-2 + |x|)$

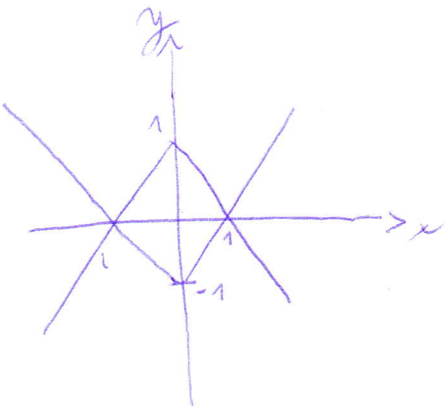


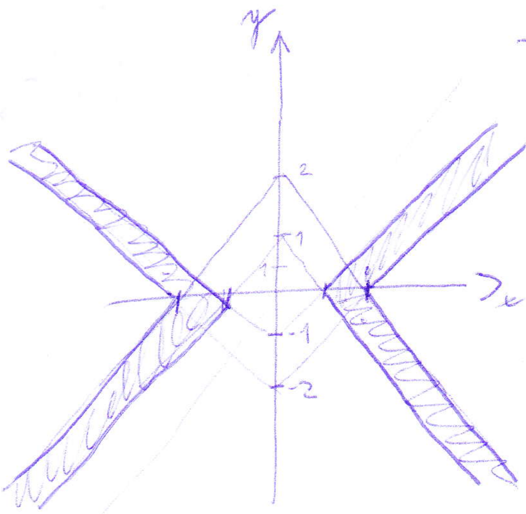
$1 \le |x| - |y|$

$1 - |x| \le -|y|$

$|x| - 1 \ge |y|$

$\pm(|x| - 1) \ge y$





→ ta osa by musela ležet cel… v ta … aby byla R…

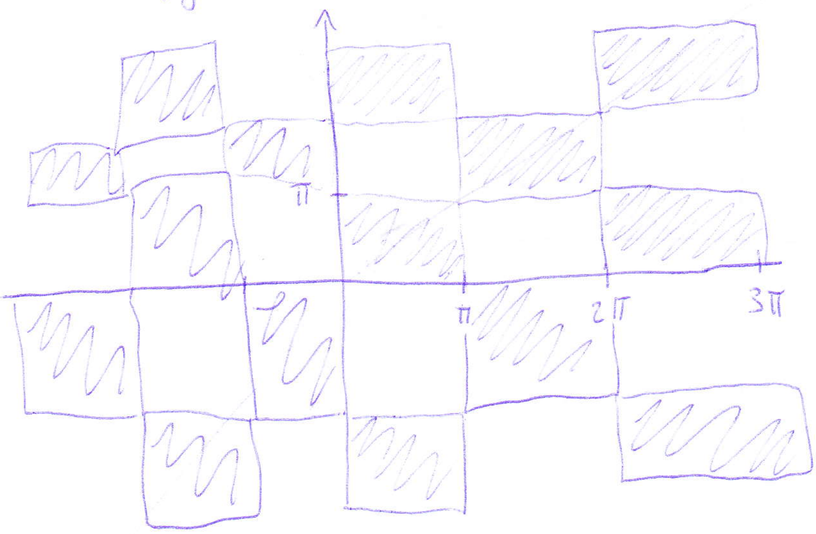
NENÍ R…

→ není sym… podle osy … to není

NENÍ SYM

$\boxed{8\diamond}$ 1)

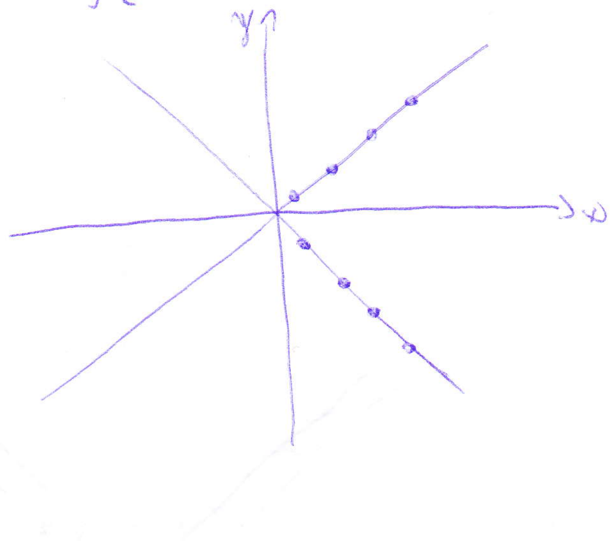
$(x,y) \in R \times R;\ \sin x \cdot \sin y \geq 0$



ora ⇒ JE REFLEXIVNÍ A SYMETRICKÁ ~~[illegible]~~

$\boxed{10\spadesuit}$ 1)

NENÍ REFLEXIVNÍ
JE SYMETRICKÝ



$3^{100} \mod 19$

$$
\begin{array}{ccccc}
81 & 243 & 729 & 2187 & 6561 \\
-76 & -228 & -722 & -2185 & 6556 \\
\hline
5 & 15 & 7 & 2 & 6
\end{array}
$$

$$
\begin{array}{cc}
19683 & 59049 \\
-19665 & -59033 \\
\hline
 & \boxed{16}
\end{array}
$$

$3^1 \equiv 3 \mod 19$

$3^2 \equiv 9 \mod 19$

$3^3 \equiv 8 \mod 19$

$3^4 \equiv 5 \mod 19$

$3^5 \equiv 15 \mod 19$

$3^6 \equiv 7 \mod 19$

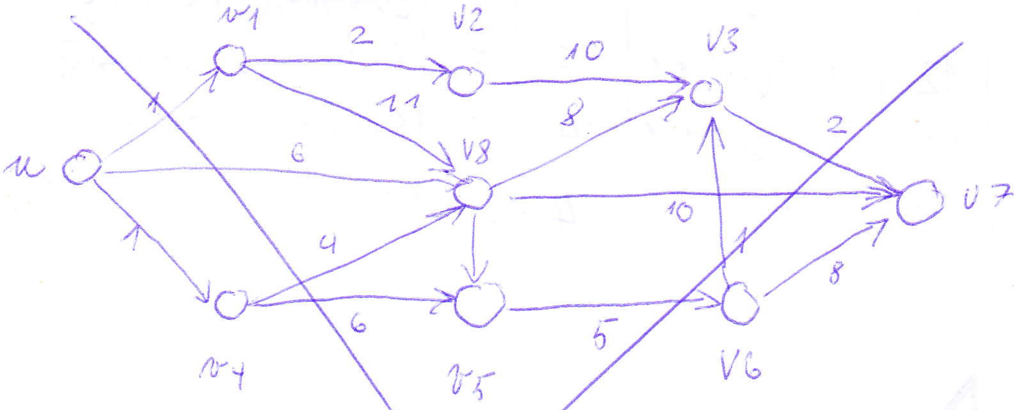
$3^7 \equiv 2 \mod 19$

$3^8 \equiv 6 \mod 19$

$3^9 \equiv 18 \mod 19 \rightarrow -1 \mod 19$

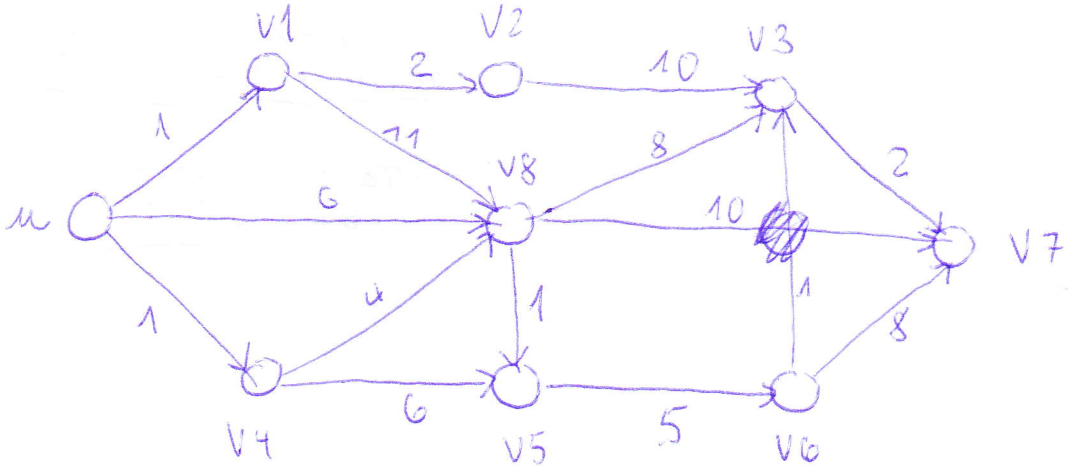
$(3^9)^{10} \cdot 3^{10} = (-1)^{10} \cdot 3^{10} \mod 19 = 16$

$3^{10} \mod 19 \Rightarrow 16$

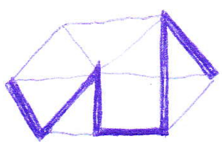


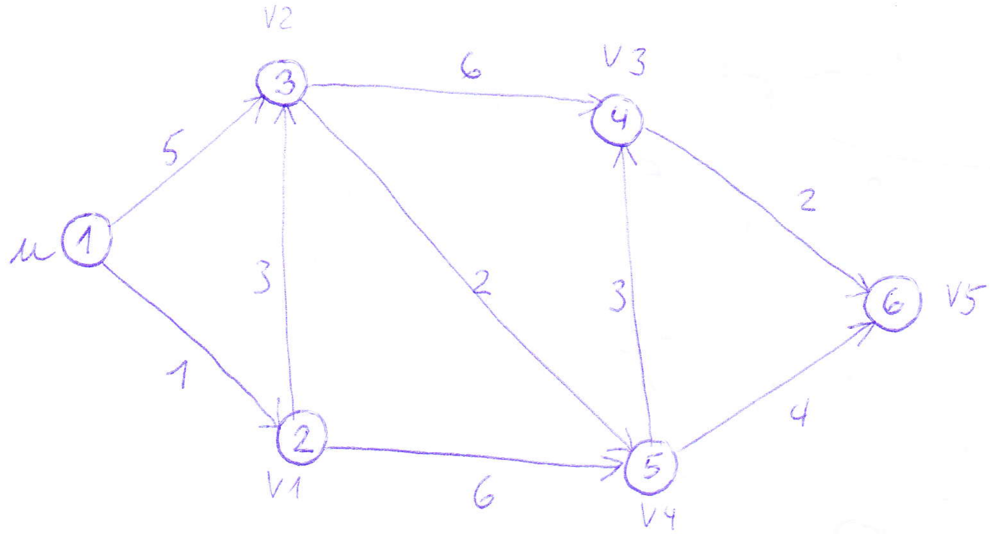
z u do V7

u	v1	v2	v3	v4	v5	v6	v7	v8
0	∞	∞	∞	∞	∞	∞	∞	6
	1	∞	∞	1	∞	∞	∞	6
	1	3	∞	1	∞	∞	∞	

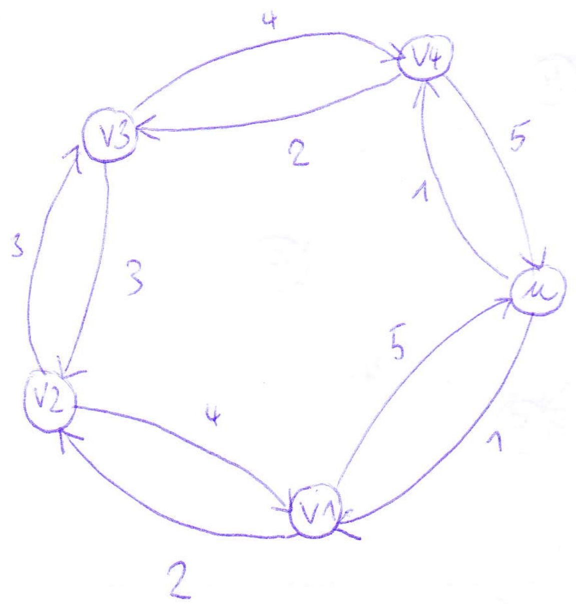


I	V'	V1	V2	V3	V4	V5	V6	V7	V8	T
0	-	1	∞	∞	1	∞	∞	∞	6	{u}
1	V1	1	3	∞	1	∞	∞	∞	6	{u, V1}
2	V4	1	3	∞	1	7	∞	∞	5	{u, V1, V4}
3	V2	1	3	13	1	7	∞	∞	5	{u, V1, V4, V2}
4	V8	1	3	13	1	6	∞	15	5	{u, V1, V4, V2, V8}
5	V5	1	3	13	1	6	11	15	5	{U, V1, V4, V2, V8, V5}
6	V6	1	3	12	1	6	11	15	5	{U, V1, V4, V2, V8, V5, V6}
7	V3	1	3	12	1	6	11	14	5	{U, V1, V4, V2, V8, V5, V6
8	V7	1	3	12	1	6	11	14	5	





I	V'	V1	V2	V3	V4	V5		
0	u	1	5	∞	∞	∞		{u}
1	V1	1	4	∞	7	∞		u, V1
2	V2	1	4	10	6	∞		u, V1, V2
3	V4	1	4	9	6	10		u, V1, V2, V4
4	V3	1	4	9	6	10		u, V1, V2, V4, V3
5	V5	1	4	9	6	10		u, V1, V2, V4, V3, V5



I	V'	V1	V2	V3	V4		
0	u	1	∞	∞	4		u
1	V1	1	3	∞	1		u, V1
2	V4	1	3	3	1		u, V1, V4
3	V2	1	3	3	1		u, V1, V4, V2
4	V3	1	3	3	1		u, V1, V4, V2

$\boxed{10 ♠}$ 2)

$5^{60} \bmod 19$

$5^1 \equiv 5 \bmod 19$
$5^2 \equiv 6 \bmod 19$
$5^3 \equiv 11 \bmod 19$
$5^4 \equiv 17 \bmod 19 \rightarrow -2 \bmod 19$
$5^5 \equiv 9 \bmod 19$
$5^6 \equiv 7 \bmod 19$
$5^7 \equiv 16 \bmod 19$
$5^8 \equiv 4 \bmod 19$
$5^9 \equiv 1 \bmod 19$
$\cancel{5^{10} \equiv 5 \bmod 19}$
$\cancel{5^{11} \equiv 6 \bmod 19}$

$$\begin{array}{r}125\\-114\\\hline 11\end{array}\quad\begin{array}{r}625\\-608\\\hline 17\end{array}\quad\begin{array}{r}3125\\-3116\\\hline 9\end{array}\quad\begin{array}{r}15625\\-15618\\\hline 7\end{array}$$

$$\begin{array}{r}78125\\-78109\\\hline 16\end{array}\quad\begin{array}{r}390625\\-390621\\\hline 4\end{array}\quad\begin{array}{r}1953125\\-1953124\\\hline\end{array}$$

$$\begin{array}{r}9765625\\-9765620\\\hline 5\end{array}\quad\begin{array}{r}48828125\\-48828119\\\hline 6\end{array}$$

9765625
1953125
−195

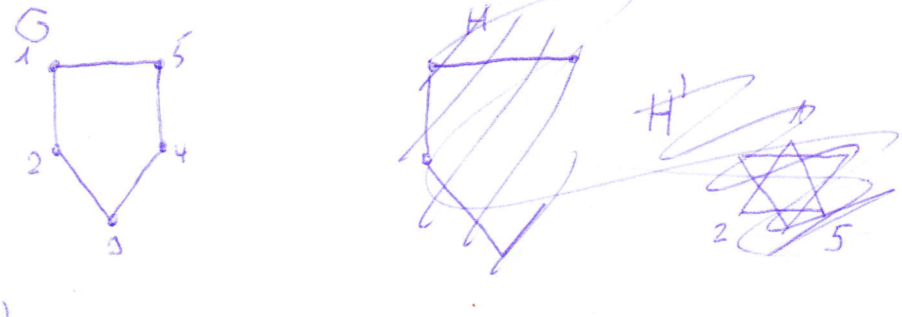
$1 \bmod 19 = -18 \quad (-2^{2})^{6} \quad (5^4)^{14} \cdot 5^4 = (-2)^{14}$

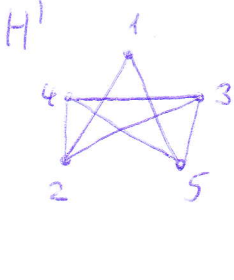
$(-18)^9$

$(5^9)^6 \cdot 5^6 \cancel{\bmod 19} = 1 \cdot 7 \bmod 19 \Rightarrow \underline{\underline{7}}$

---

homomorfismus





← najdu v ~~to~~ G, ale v G už nenajdu H'

příklad homomorfism

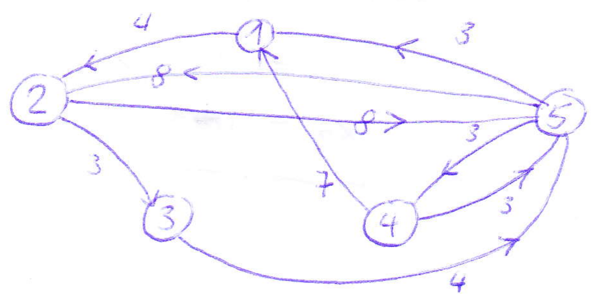
$\boxed{5♡}$ 2)

$$\begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 3 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 3 & 1 \end{vmatrix}_{\text{mod } 5} \begin{matrix} \\ +2\mathrm{I} \\ +\mathrm{I} \\ +2\mathrm{I} \\ \\ \end{matrix} = \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 3 & 0 & 3 & 3 \\ 0 & 2 & 2 & 2 & 2 \\ 0 & 3 & 2 & 1 & 3 \\ 0 & 1 & 1 & 3 & 1 \end{vmatrix}_{\text{mod } 5}$$

$$= \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 1 \\ 0 & 2 & 2 & 2 & 2 \\ 0 & 3 & 2 & 1 & 3 \\ 0 & 3 & 0 & 3 & 3 \end{vmatrix}_{\text{mod } 5} \begin{matrix} \\ \\ +3\mathrm{II} \\ +2\mathrm{II} \\ +2\mathrm{II} \end{matrix} = \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 2 & 4 & 0 \end{vmatrix}_{\text{mod } 5} =$$

$$= \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 1 \\ 0 & 0 & 2 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & & & & \end{vmatrix}$$

$$D_0 = \begin{bmatrix} 0 & 4 & \infty & \infty & \infty \\ \infty & 0 & 3 & \infty & 8 \\ \infty & \infty & 0 & \infty & 4 \\ 7 & \infty & \infty & 0 & 3 \\ 3 & 8 & \infty & 3 & 0 \end{bmatrix}$$



$$D_1 = \begin{bmatrix} 0 & 4 & \infty & \infty & \infty \\ \infty & 0 & 3 & \infty & 8 \\ \infty & \infty & 0 & \infty & 4 \\ 7 & \infty & \infty & 0 & 3 \\ 3 & 8 & \infty & 3 & 0 \end{bmatrix} \boxdot \begin{bmatrix} 0 & 4 & \infty & \infty & \infty \\ \infty & 0 & 3 & \infty & 8 \\ \infty & \infty & 0 & \infty & 4 \\ 7 & \infty & \infty & 0 & 3 \\ 3 & 8 & \infty & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 7 & \infty & 12 \\ 10 & 0 & 3 & 11 & 7 \\ 7 & 12 & 0 & 7 & 4 \\ 6 & 11 & \infty & 0 & 3 \\ 3 & 7 & 11 & 3 & 0 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 0 & 4 & 7 & \infty & 12 \\ 10 & 0 & 3 & 11 & 7 \\ 7 & 12 & 0 & 7 & 4 \end{bmatrix} \boxdot \begin{bmatrix} 0 & 4 & 7 & \infty & 12 \\ 10 & 0 & 3 & 11 & 7 \\ 7 & 12 & 0 & 7 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 7 & 14 & 11 \\ 10 & 0 & 3 & 10 & 7 \end{bmatrix} = D$$

$\boxed{3\heartsuit}$ 2)

$$\left(\begin{array}{ccccc|c} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 & 1 & 2 \\ 1 & 2 & 2 & 3 & 2 & 1 \\ 2 & 1 & 2 & 2 & 2 & 3 \end{array}\right) \begin{array}{l} \text{mod } 5 \\ +3I \\ +4I \\ +3I \end{array} \sim \left(\begin{array}{ccccc|c} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 0 & 4 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \text{mod } 5 \\ +2II \\ \\ + \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 4 & 0 & 4 & 2 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \text{mod } 5 \\ \\ \\ +3II \end{array} \sim \left(\begin{array}{ccccc|c} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 4 & 0 & 4 & 2 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 & 2 & 2 \end{array}\right) \begin{array}{l} \\ \\ \\ +3III \end{array} \sim$$

$$\left(\begin{array}{ccccc|c} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 4 & 0 & 4 & 2 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 \end{array}\right) \Rightarrow hod(A) = hod(A|b) \Rightarrow \text{má řeš}$$

$\boxed{x_4 = 2}$

$\cancel{x_2 + 4 - 4q} + 4q$

$\cancel{x_2 = 1}$

$\cancel{x_2 + 4 - 4q + 4q = 2}$

$\boxed{x_5 = q}$

$x_3 + 4 + q = 0$

$x_3 = -4 + q$

$\boxed{x_3 = 1 - q}$

$x_2 = -2$

$\Rightarrow \boxed{x_2 = 3}$

$x_1 + 6 + 1 - q + 2 + q = 1$

$x_1 = {+8} \Rightarrow \boxed{x_1 = 2}$

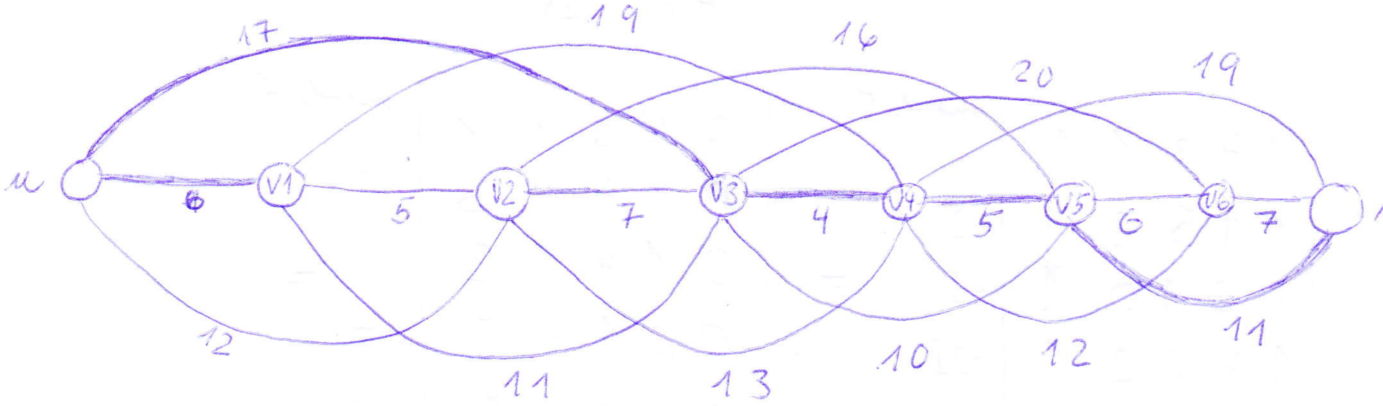
$$x = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \\ 0 \end{bmatrix} + q \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \\ 1 \end{bmatrix}$$

$\cancel{x_3 + 4 + q} =$

$x_3 + 4 + q = 0$

$x_3 + x_5 = 1$

$x_3 + q = 1$



I	v	V1	V2	V3	V4	V5	V6		v
0	u	6	12	17	∞	∞	∞	~~u~~	∞
1	V1	6	11	17	25	∞	∞	u, V1	∞
2	V2	6	11	17	24	27	∞	~~u, V1, V2~~	∞
3	V3	6	11	17	21	27	37	~~u, V1, V3~~	∞
4	V4	6	11	17	21	26	33		40
5	V5	6	11	17	21	26	32		37
6	V6	6	11	17	21	26	32		37
7	v	6	11	17	21	26	32		37

min. cesta do v je 37

$\boxed{8 \heartsuit}$ 2)

$$\left(\begin{array}{ccccc|c} 1 & 2 & 3 & 1 & 1 & 2 \\ 2 & 1 & 3 & 2 & 1 & 3 \\ 1 & 2 & 3 & 3 & 2 & 4 \\ 2 & 1 & 3 & 2 & 2 & 3 \end{array}\right)^{\text{mod } 5} \begin{array}{l} \\ +3\mathrm{I} \\ +4\mathrm{I} \\ +3\mathrm{I} \end{array} \sim \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 1 & 1 & 2 \\ 0 & 2 & 2 & 0 & 4 & 4 \\ 0 & 0 & 0 & 2 & 1 & 2 \\ 0 & 2 & 2 & 0 & 0 & 4 \end{array}\right)^{\text{mod } 5} \begin{array}{l} \\ -\mathrm{IV} \\ \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 & 1 & 2 \\ 0 & 2 & 2 & 0 & 0 & 4 \end{array}\right)^{\text{mod } 5} \sim \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 1 & 1 & 2 \\ 0 & 2 & 2 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 4 & 0 \end{array}\right)^{\text{mod } 5}$$

$\underline{x_5 = 0}$

$x_3 = q$

$2x_2 + 2q = 4$

$2x_4 + 0 = 2$

$\underline{x_4 = 1}$

$\cancel{x_2 = 4 - 2q}$ $\quad \underline{x_2 = 2 - q}$

$x_1 + 2 - q + 3q + 1 + 0 = 2$

$x_1 = 4q + q$

$\underline{x_1 = 2 + q}$

$$\cancel{x = \begin{bmatrix} 1 \\ 4 \\ 0 \\ 2 \\ 0 \end{bmatrix} + q \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}}$$

$$x = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + q \begin{bmatrix} 4 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$q \in \mathbb{Z}_5 (0, 1, 2, 3, 4)$

$\boxed{J\heartsuit}$ 2)

$3^{100} \bmod 17$

$$\begin{array}{r} 81 \\ -68 \\ \hline 13 \end{array} \quad \begin{array}{r} 243 \\ -238 \\ \hline 5 \end{array} \quad \begin{array}{r} 729 \\ -714 \\ \hline 15 \end{array} \quad \begin{array}{r} 2187 \\ -2176 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 6561 \\ -6545 \\ \hline 16 \end{array}$$

$3^1 \equiv 3 \bmod 17$

$3^2 \equiv 9 \bmod 17$

$3^3 \equiv 10 \bmod 17$

$3^4 \equiv 13 \bmod 17$

$3^5 \equiv 5 \bmod 17$

$3^6 \equiv 15 \bmod 17$

$3^7 \equiv 11 \bmod 17$

$3^8 \equiv 16 \bmod 17 \longrightarrow \underline{-1 \bmod 17}$

$(3^8)^{12} \cdot 3^4 = (-1)^{12} \cdot 3^4 = 1 \cdot 13 \bmod 17 \Rightarrow \underline{\underline{\text{zbytek} = 13}}$

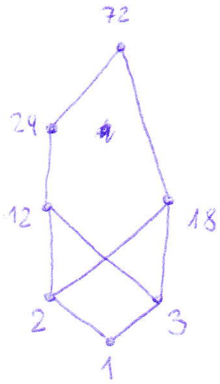
$\boxed{K\heartsuit}$ 2)

$$\begin{vmatrix} 2 & 4 & 1 & 1 & 1 \\ 1 & 1 & 3 & 2 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 1 & 4 & 0 & 2 & 1 \\ 0 & 1 & 3 & 3 & 1 \end{vmatrix} \begin{array}{l} \bmod 5 \\ +2\mathrm{I} \\ +\mathrm{I} \\ +2\mathrm{I} \\ \end{array} = \begin{vmatrix} 2 & 4 & 1 & 1 & 1 \\ 0 & 4 & 0 & 4 & 3 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 2 & 2 & 4 & 3 \\ 0 & 1 & 3 & 3 & 1 \end{vmatrix} \bmod 5 = (-1) \begin{vmatrix} 2 & 4 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 2 & 2 & 4 & 3 \\ 0 & 4 & 0 & 4 & 3 \end{vmatrix}$$

$$= (-1) \begin{vmatrix} 2 & 4 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 3 & 2 & 4 \end{vmatrix} \bmod 5 = 1 \cdot \begin{vmatrix} 2 & 4 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 3 & 2 & 4 \end{vmatrix} \begin{array}{l} \bmod 5 \\ \\ = \\ +3\mathrm{III} \\ +2\mathrm{III} \end{array} \begin{vmatrix} 2 & 4 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 \end{vmatrix} = \dots$$

$\boxed{A ♡}$ 1)

$M_1 = \{1, 2, 3, 12, 18, 24, 72\}$

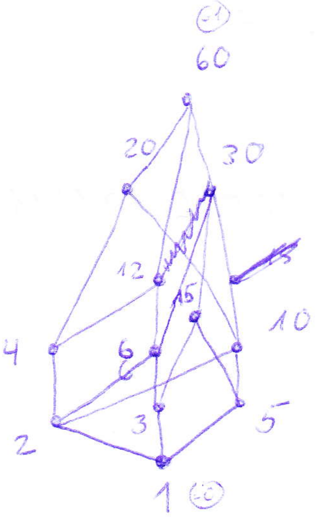


NENÍ SVAZ

pro $2 \wedge 3 \nexists$

není splněna podmínka, že pro každé dva prvky $\exists$ sup a inf

$M_2 = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$



min = 1
max = 60
sup = 60
inf = 1

pro každé dva prvky existuje sup

=> JE TO SVAZ

distributivní ?

Nenalezen žádný zakázaný podsvaz M5, N5

ANO

komplementární ?

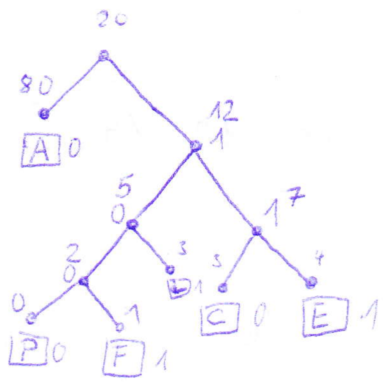
$\overline{1} = 60$
$\overline{60} = 1$
$\overline{2} = 30$
$\overline{30} = 2$
$\overline{3} = 20$
$\overline{5} = 12$
$\overline{12} = 5$

$\overline{4} = 15$
$\overline{15} = 4$
$\overline{10} = 6$
$\overline{6} = 10$

ANO

=> ke každému prvku existuje komplement

A - 8	A - 8	A - 8	A - 8	A - 8	AECLPF -
E - 4	E - 4	E - 4	EC - 7	ECLPF - 12	
C - 3	C - 3	C - 3	LPF - 5		
L - 3	L - 3	LPF - 5			
P - 1	PF - 2				
F - 1					



A - 0
B - 1000
C - 110
F - 1001
L - 101
E - 111

PALEC = 1000|0|101|111|110

grafová ppst

7 7 6 6 5 5 5 3 2 2

6 5 5 4 4 4 2 2 2

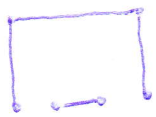
4 4 3 3 3 1 2 2

4 4 3 3 3 2 2 1

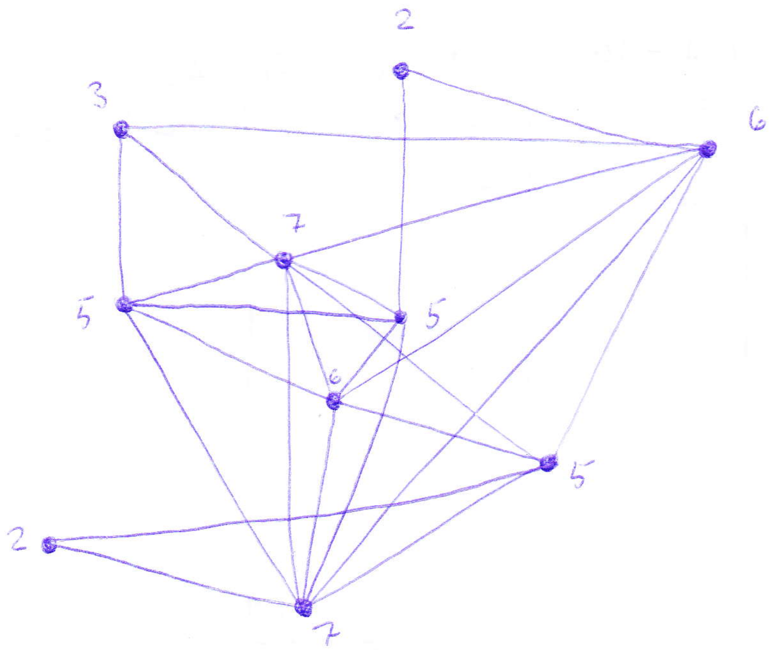
3 2 2 2 2 2 1

1 1 1 2 2 1

2 2 1 1 1 1 ... umím nakreslit

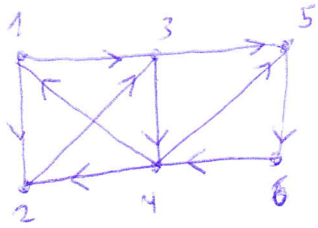


=> JE TO GRAFOVÁ PPST



počet koster

$$L = \begin{pmatrix} 3 & -1 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 & 0 \\ -1 & -1 & 4 & -1 & -1 & 0 \\ -1 & -1 & -1 & 5 & -1 & -1 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{pmatrix}$$



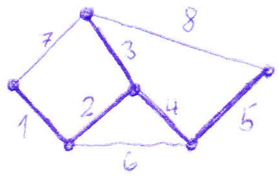
$$\det \begin{vmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 5 & -1 \\ 0 & 0 & -1 & -1 & 3 \end{vmatrix} \begin{array}{l} \\ \\ \\ -\mathrm{III} \\ +3\mathrm{III} \end{array} = \begin{vmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & -5 & 6 & 0 \\ -3 & -3 & 11 & -4 & 0 \end{vmatrix} =$$

$$= (-1) \cdot \begin{vmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & 0 & -5 & 6 \\ -3 & -3 & 11 & -4 \end{vmatrix} \begin{array}{l} +3\mathrm{II} \\ \\ \\ -3\mathrm{II} \end{array} = (-1) \cdot \begin{vmatrix} 0 & 8 & -4 & -4 \\ -1 & 3 & -1 & -1 \\ 0 & 0 & -5 & 6 \\ 0 & -12 & 14 & -1 \end{vmatrix} =$$

$$= (-1) \begin{vmatrix} 8 & -4 & -4 \\ 0 & -5 & 6 \\ -12 & 14 & -1 \end{vmatrix} = (-1) \cdot \left[(40 + 0 + 288) - (-240 + 0 + 672) \right] =$$

$$= -(-1) \cdot \left[328 - 432 \right] = \underline{\underline{104}}$$

určení prostoru kružnic



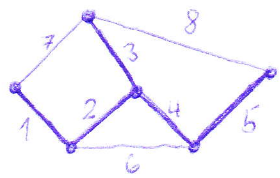
$$\cancel{\begin{array}{c|cccc|ccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ C_6 = & & & & & & & \\ C_7 = & & & & & & & \\ C_8 = & & & & & & & \end{array}}$$

$$\begin{array}{l} \begin{array}{ccccc|ccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ C_6 = \left[\begin{array}{ccccc|ccc} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right] \\ C_7 = \left[\begin{array}{ccccc|ccc} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \\ C_8 = \left[\begin{array}{ccccc|ccc} 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \end{array} \Rightarrow K_F = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

určení prostoru řezů

$$\begin{array}{l} R_1 = \left[\begin{array}{ccccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \\ R_2 = \left[\begin{array}{ccccc|ccc} 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \\ R_3 = \left[\begin{array}{ccccc|ccc} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{array}\right] \\ R_4 = \left[\begin{array}{ccccc|ccc} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right] \\ R_5 = \left[\begin{array}{ccccc|ccc} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \end{array} \Rightarrow Q_F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

určení prostoru kružnic



$$
\begin{array}{c c}
 & \begin{array}{ccccc|ccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
C_6 = & \left[\begin{array}{ccccc|ccc} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right] \\
C_7 = & \left[\begin{array}{ccccc|ccc} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \\
C_8 = & \left[\begin{array}{ccccc|ccc} 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]
\end{array}
\Rightarrow K_F = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}
$$

určení prostoru řezů

$$
\begin{array}{c c}
R_1 = & \left[\begin{array}{ccccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \\
R_2 = & \left[\begin{array}{ccccc|ccc} 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \\
R_3 = & \left[\begin{array}{ccccc|ccc} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{array}\right] \\
R_4 = & \left[\begin{array}{ccccc|ccc} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right] \\
R_5 = & \left[\begin{array}{ccccc|ccc} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]
\end{array}
\Rightarrow Q_F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}
$$