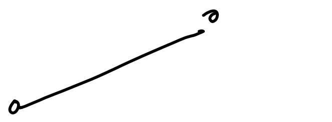
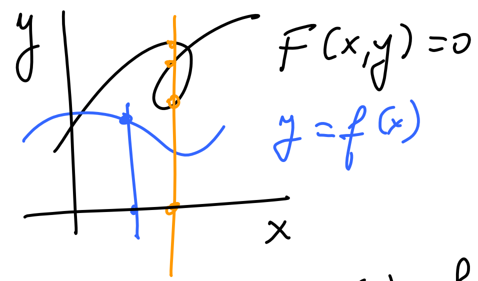
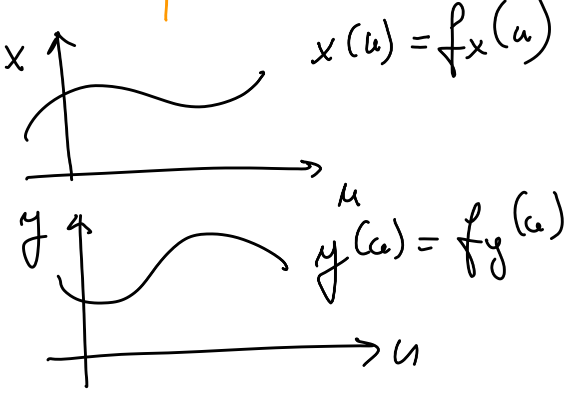
ZPG - 8

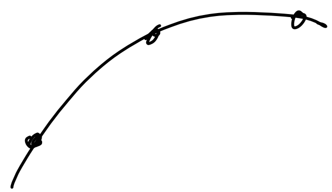
Parametrické vyjádření křivek

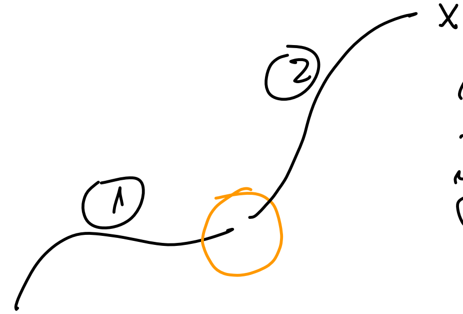
- lineární interpolace
- kvadratické interpolace
- kubiky

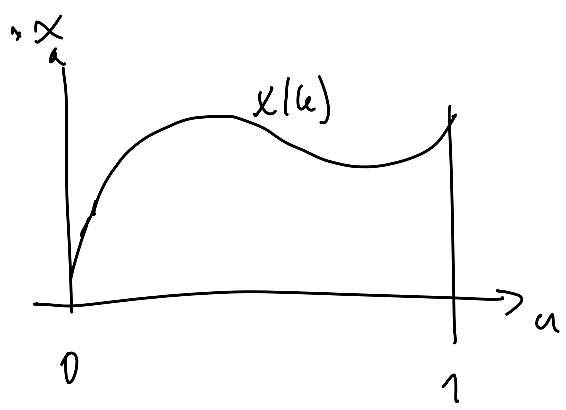












$$x(u) = a_x u^3 + b_x u^2 + c_x u + d_x$$

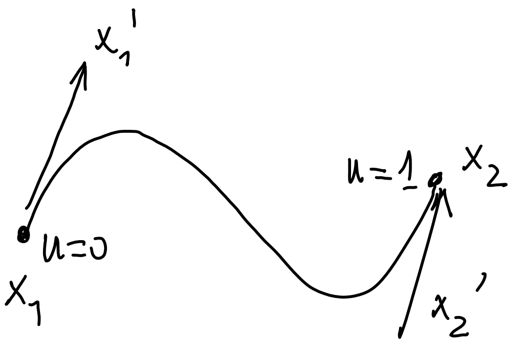
$$y(u) = a_y u^3 + b_y u^2 + c_y u + d_y$$

$$x(u) = au^3 + bu^2 + cu + d$$

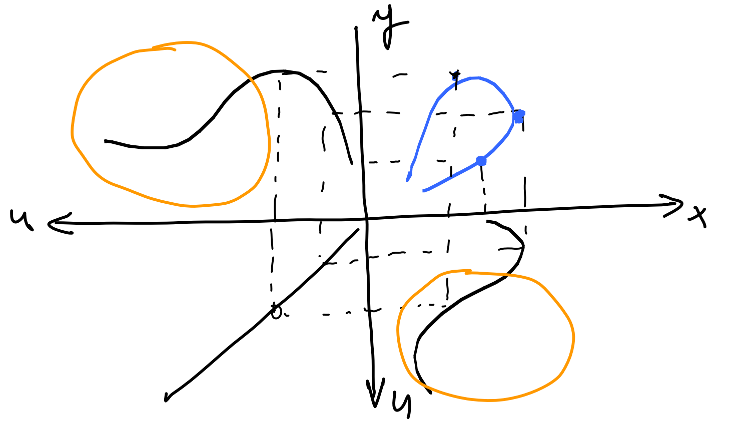
Hermitova forma

$$x = [\;\cdots\;]\, M_f \begin{vmatrix} u^3 \\ u^2 \\ u \\ 1 \end{vmatrix}$$

M_H M_B



$$x_1' = \left.\frac{dx}{du}\right|_{x=x_1}$$



$$x' = 3au^2 + 2bu + c$$

$$x_1 = d$$
$$x_2 = a + b + c + d$$
$$x_1' = c$$
$$x_2' = 3a + 2b + c$$

Známe

$a, b, c, d = ?$

$$\mathbb{A}x = b$$

$[a, b, c, d]^T$

$[x_1, x_2, x_1', x_2']^T$

$$\mathbb{A}x = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ e \\ d \end{bmatrix}$$

?

Řešením $Ax=b$ dostáváme

$$a = 2x_1 - 2x_2 + x_1' + x_2'$$
$$b = -3x_1 + 3x_2 - 2x_1' - x_2'$$
$$c = x_1'$$
$$d = x_1$$

$$x(u) = au^3 + bu^2 + cu + d$$

$$x(u) = (2x_1 - 2x_2 + x_1' + x_2').u^3 + (-3x_1 + 3x_2 - 2x_1' - x_2').u^2 + \dots$$

$$\Downarrow \quad x(u) = (\quad).x_1 + (\quad)x_2 + (\,)x_1' + (\,)x_2'$$

upravou

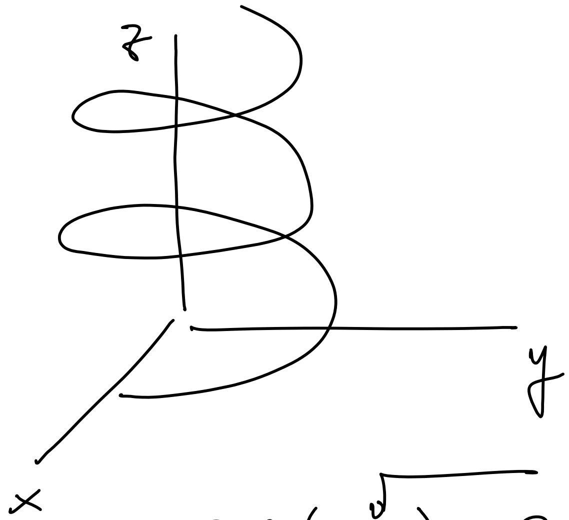
$$x(u) = (2u^3 - 3u^2 + 1)x_1 + (-2u^3 + 3u)x_2 + (u^3 - 2u^2 + u)x_1' + (u^3 - u^2).x_2'$$

$$x(u) = [x_1, x_2, x_1', x_2'] \cdot \begin{bmatrix} 2u^3 - 3u^2 + 1 \\ -2u^3 + 3u \\ u^3 - 2u^2 + u \\ u^3 - u^2 \end{bmatrix} = [x_1\ x_2\ x_1'\ x_2'] \cdot \begin{bmatrix} 2 & -3 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{bmatrix}$$

$$= \underline{\mathbf{x}^T . M_H . \mathbf{u}}$$

Nepamatovat!

$$\begin{aligned} x(u) &= \mathbf{x}^T M_H \mathbf{u} \\ y(u) &= \mathbf{y}^T M_H \mathbf{u} \\ z(u) &= \mathbf{z}^T M_H \mathbf{u} \end{aligned}$$



Příklad

$$x(u) = r \cdot \cos\varphi$$
$$y(u) = r \cdot \sin\varphi$$
$$z(u) = \varphi$$

! Vypočítej schéma !

$$X(u) = X^T M_f u \qquad y(u) \ldots, \quad z(u) \ldots$$

CUBIC ($x_1\, x_2\, x_1'\, x_2'$ & $y_1\, y_2\, y_1'\, y_2'$)

shift TO (x_1, y_1)

for $u = 0$ to 1 step 0.01 konst.

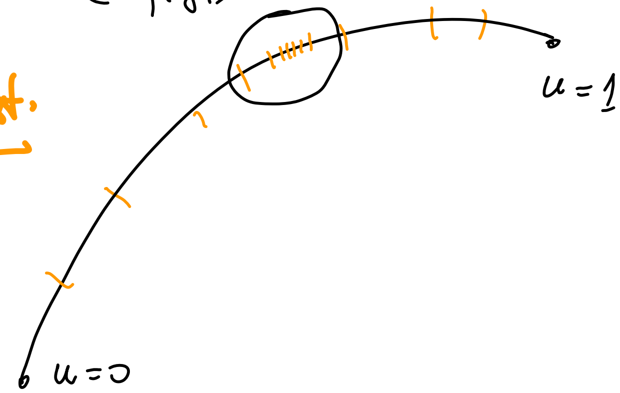
{

$x_p = x^T M_f u$

$y_p = y^T M_f u$

DRAW TO (x_p, y_p)

}



$xx = x^T M_f \; ; \; yy = y^T \cdot M_f$

for

{

$x_p = xx^T u \; ; \; y_p = yy \cdot u$ $\quad [u^3, u^2, u, 1]$

DRAWTO(x_p, y_p)

}

výpočet polynomu

$x_p = \sum xx_i u_i$!!

u^2, u^3

$$P(x) = \sum_{i=0}^{n} a_i x^i$$

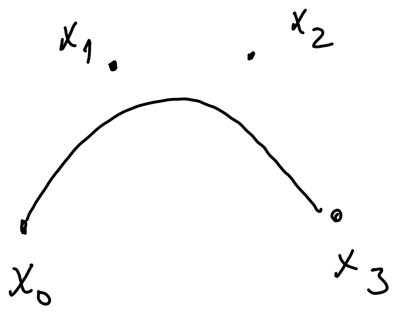
$$a_0 + a_1 x + a_2 x^2 + \dots a_n x^n + a_k x^k$$

$$= a_k x^k + a_{k-1} x^{k-1} + \dots + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$= ((\dots((a_k x + a_{k-1}) x + a_{k-2}) \cdot x + \dots a_2) x + a_1) x + a_0$$

Hornerovo schéma

Bezier



$$x(u) = \sum_{i=1}^{n} B_{n_i}(u) \cdot x_i \qquad u \in \langle 0, 1 \rangle$$

$\hookrightarrow$ Bernsteinovy polynomy

$$B_{n_i} = \binom{n}{i} u^i (1-u)^{n-i}$$

$$\binom{n}{i} = \frac{n!}{i!(n-i)!} \quad \& \quad 0! = 1$$

$$x(u) = (1-u^3) x_0 + 3u(1-u)^2 x_1 + 3u^2(1-u) x_2 + u^3 x_3$$

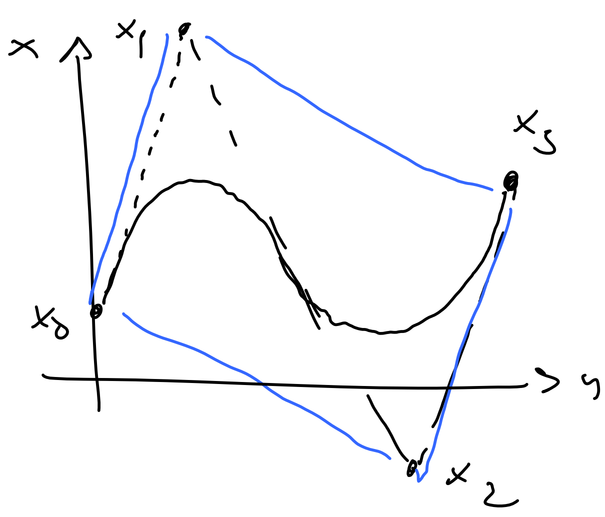
$$x(u) = [x_0, x_1, x_2, x_3] \cdot \begin{bmatrix} 1-u^3 \\ 3u(1-u)^2 \\ u^2(1-u) \\ u^3 \end{bmatrix} = [x_0\ x_1\ x_2\ x_3] \begin{vmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} u^3 \\ u^2 \\ u \\ 1 \end{vmatrix}$$

$$x^T M_B u$$

Vlastnosti Bezierovy křivky

- je vždy uvnitř konvexní obálky
- ${}^{H}x_1' = 3(x_1 - x_0)$ ${}^{H}x_2' = 3(x_3 - x_2)$

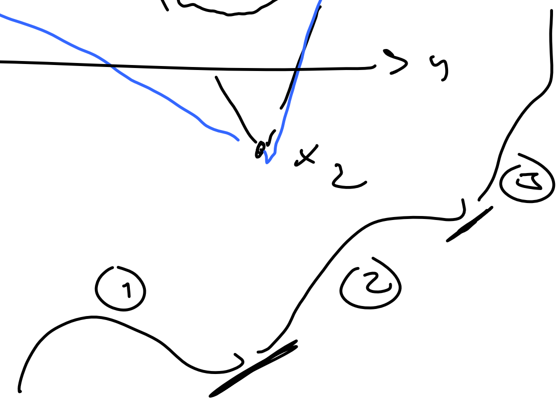
↑ Hermitova forma



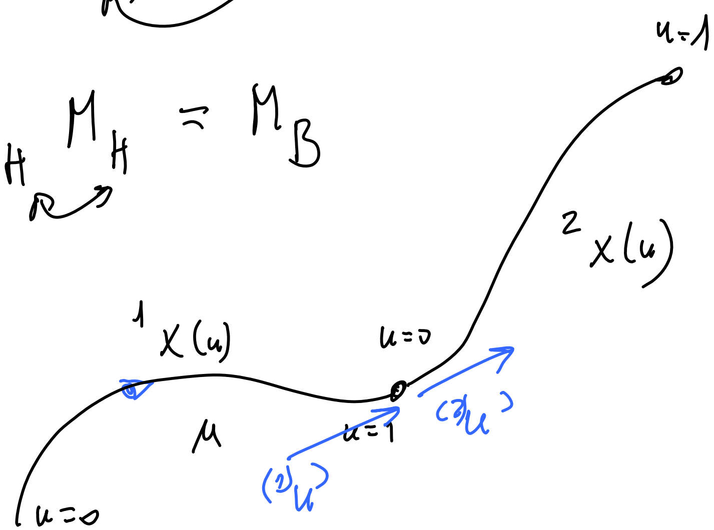
Převod

Hermit ⇄ Bezier

$$M_{BH} M_H = M_B$$



Plátování / napojování

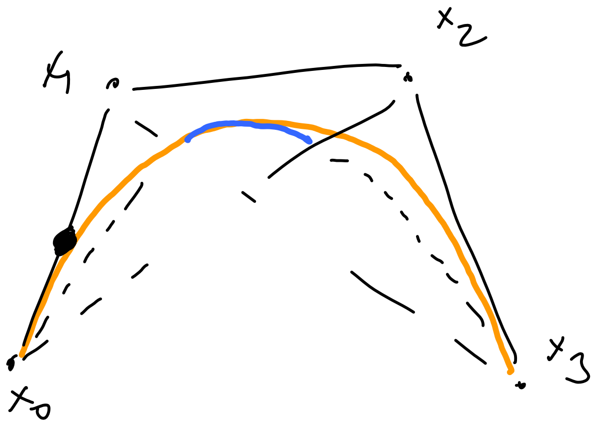


$^1x(1) = {}^2x(0)$ spojitost

$^1x'(1) = {}^2x'(0)$ hladkost C^1

$\Rightarrow Ax = b$ C^2

Coonsova kubika



$$X = [x_0, x_1, x_2, x_3]\,.\, f_C \qquad x(u) = X^T . M_f\, u$$

$$f_C = \frac{1}{6} \underbrace{\begin{vmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 0 & 4 \\ -3 & 3 & 3 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix}}_{M_C} \begin{vmatrix} u^3 \\ u^2 \\ u \\ 1 \end{vmatrix}$$

C^2

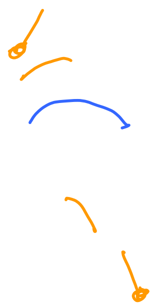
$\cancel{Ax = b}$

Uzavřená křivka

$[x_0, x_1, x_2, x_3]$, $[x_1, x_2, x_3, x_4]$, ... $[x_{n-2}, x_{n-1}, x_0, x_1]$

Dotažení do koncových bodů

$x_0\ x_0\ x_0\ x_1$
$x_0\ x_0\ x_1\ x_2$
$x_0\ x_1\ x_2\ x_3$
$x_1\ x_2\ x_3\ x_3$
$x_2\ x_3\ x_3\ x_3$



křivky

skript 33

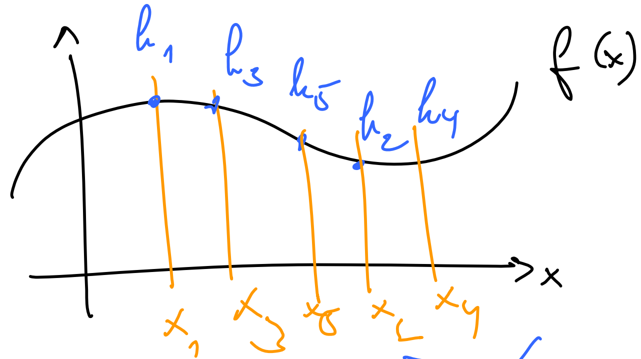
Geom. modelování

Interpolace RBF

Radiální bázové funkce

$$f(x) = \sum_{i=1}^{n} \lambda_i \phi_i(r) \qquad \text{pro } n \text{ bodů}$$

$$r = \| x - x_i \| \qquad h$$



neuspořádané

$$\phi(r) = r^2 \lg r$$

$\vdots$

$$\lambda_1 \phi_{11}, \lambda_2 \phi_{12} \dots \lambda_n \phi_{1n} = h_1$$

$$\lambda_1 \phi_{21} + \lambda_2 \phi_{22} + \dots + \lambda_n \phi_{2n} = h_n$$

$\vdots$

$$\lambda_n \phi_{n1} + \lambda_2 \phi_{n2} + \dots \lambda_n \phi_{nn} = h_n$$

$$\phi_{ij} = \phi(\underbrace{\| x_i - x_j \|}_{r})$$

$$\uparrow$$

$$r^2 \lg r$$

$$\mathbb{A} x = b \rightarrow [h_1 \dots h_n]$$

$$\hookrightarrow [\lambda_1 \dots \lambda_n]$$

Stabilita & others

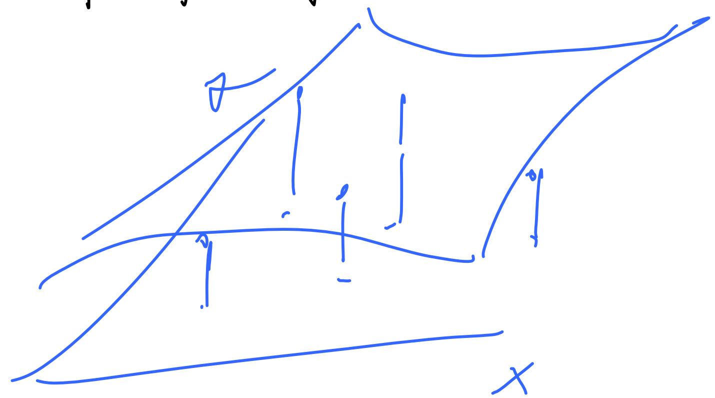
$$f(x) = \sum \lambda_i \phi_i + p(x) \quad ; \quad \sum \lambda_i = 0, \quad \sum \lambda_i x_i = 0 \quad (1)$$

$$\left(\begin{array}{ccc|ccc} & \phi_{ij} & & x_1 & y_1 & 1 \\ & & & \vdots & & \\ & & & x_n & y_n & 1 \\ \hline x_1 & \cdots & x_n & 0 & 0 & 0 \\ y_1 & \cdots & y_n & 0 & 0 & 0 \\ 1 & \cdots & 1 & 0 & 0 & 0 \end{array}\right) \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ a \\ b \\ c \end{pmatrix} = \begin{pmatrix} h_1 \\ \vdots \\ h_n \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

(1)

$$p(x) = ax + c$$

$+ by$



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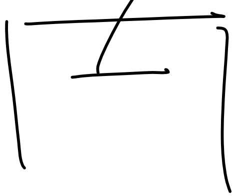
↳ MSc. thesis

↳ 

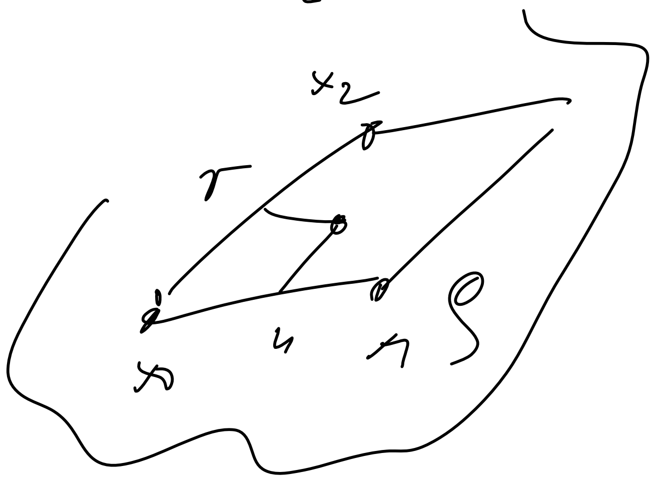
PdF

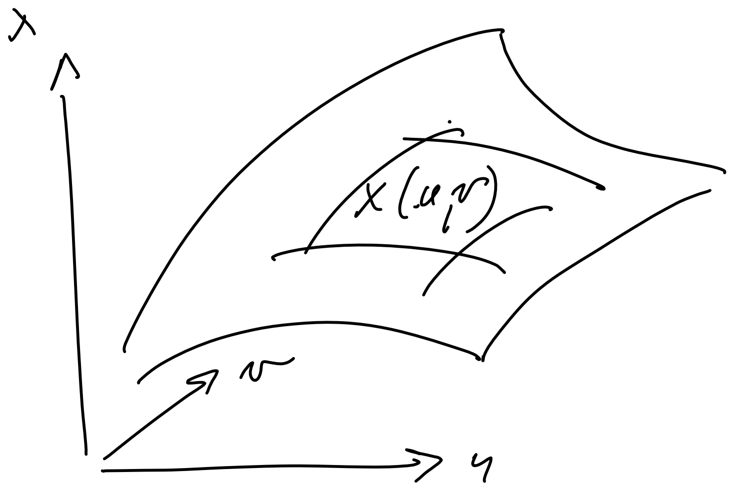
Uhlír

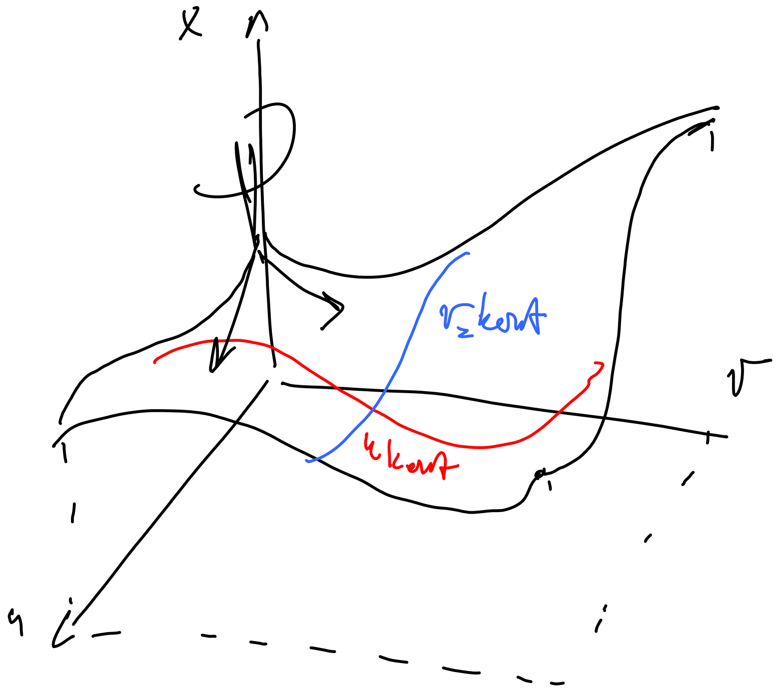
Zapletal

Plochy

$$\left.\begin{aligned} x(u) &= x_0 + (x_1 - x_0)\cdot u \\ x(u,v) &= x_0 + (x_1 - x_0)\cdot u \\ &\quad + (x_2 - x_0)\cdot v \end{aligned}\right\} \text{lin}$$







$x(u,v)$ $\quad$ $y(u,v)$, $z(u,v)$

! $x(u, v = \text{konst})$ -- kubika $\}$ Hermit

! $x(u = \text{konst}, v)$ -- kubika $\}$ Bezier

$\equiv$

$x(u) = \mathbb{x}^T \mathbb{M}_f \mathbb{u}$

$x(v) = \mathbb{x}^T \mathbb{M}_f \mathbb{v}$

$x(u = \text{konst}, v) = \mathbb{x}^T_{u=\text{konst}} \mathbb{M}_f \cdot \mathbb{v}$

$$\boxed{x(u, v) = \mathbb{u}^T \mathbb{M}_f^T \mathbb{X} \mathbb{M}_f \mathbb{v}}$$

řídící body plochy

16 $\quad 2 = 4 \times 4$