Příklad : Hopfieldova síť (synchronní model)

Dány trénovací vektory $a^1 = [1\ 1\ 1\ 0]^T$,
$a^2 = [1\ 1\ 0\ 0]^T$

$w_{ij} = \sum_{s=1}^{P}(2a_i^s - 1)(2a_j^s - 1)\,,\ i \neq j,\ \ 1 \leq i,j \leq N\,,$ tj.

$$\mathbf{W} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 & -2 \\ 2 & 2 & 0 & -2 \\ 0 & 0 & 2 & 0 \\ -2 & -2 & 0 & 2 \end{bmatrix}$$

$\mathbf{\Theta} = \frac{1}{2}\sum_{j=1}^{M} w_{ij} = [\ 1\ \ 1\ \ 1\ \ -1]^T$

Pro testovací vektor $\mathbf{y}(0) = \mathbf{x} = [\ 0\ \ 1\ \ 0\ \ 0\ ]^T$ dostaneme:

a) v 1. iteraci:

$$\mathbf{Wx} - \mathbf{\Theta} = \begin{bmatrix} 2 & 2 & 0 & 2 \\ 2 & 2 & 0 & -2 \\ 0 & 0 & 2 & 0 \\ -2 & -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\mathbf{y}(1) = f_H(\mathbf{Wx} - \mathbf{\Theta}) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

b) v 2. iteraci:

$$\mathbf{Wx} - \mathbf{\Theta} = \begin{bmatrix} 2 & 2 & 0 & 2 \\ 2 & 2 & 0 & -2 \\ 0 & 0 & 2 & 0 \\ -2 & -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -1 \\ -3 \end{bmatrix}$$

$$\mathbf{y}(2) = f_H(\mathbf{Wx} - \mathbf{\Theta}) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Příklad : Hopfieldova síť (asynchronní model)

Dány trénovací vektory $a^1 = [1\ 1\ 1\ 0]^T$,
$a^2 = [1\ 1\ 0\ 0]^T$

$$w_{ij} = \begin{cases} \sum_{s=1}^{P}(2a_i^s - 1)(2a_j^s - 1)\,, & i \neq j,\quad 1 \leq i,j \leq N\,, \\ 0 & i = j \end{cases}\,, \quad \text{tj.}$$

$$\mathbf{W} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & -2 \\ 2 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ -2 & -2 & 0 & 0 \end{bmatrix}$$

$\mathbf{\Theta} = \frac{1}{2}\sum_{j=1}^{M} w_{ij} = [\,0\ \ 0\ \ 0\ \ -2]^T$

Pro testovací vektor $\mathbf{y}(0) = \mathbf{x} = [\,0\ \ 1\ \ 0\ \ 0\,]^T$ dostaneme:
a) v 1. iteraci:

$$\mathbf{y}(1) = f_H(\mathbf{W}\mathbf{x} - \mathbf{\Theta}) = \begin{bmatrix} 0 & 2 & 0 & 2 \\ 2 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ -2 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \end{bmatrix} \quad \text{postupně}$$

1. bit : $[\,1\ \ 1\ \ 0\ \ 0\,]^T$
2. bit : $[\,1\ \ 1\ \ 0\ \ 0\,]^T$
3. bit : $[\,1\ \ 1\ \ 0\ \ 0\,]^T$
4. bit : $[\,1\ \ 1\ \ 0\ \ 0\,]^T$

b) v 2. iteraci:

$$\mathbf{y}(2) = f_H(\mathbf{W}\mathbf{x} - \mathbf{\Theta}) = \begin{bmatrix} 0 & 2 & 0 & 2 \\ 2 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ -2 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \end{bmatrix} \quad \text{postupně}$$

1. bit : $[\,1\ \ 1\ \ 0\ \ 0\,]^T$
2. bit : $[\,1\ \ 1\ \ 0\ \ 0\,]^T$
3. bit : $[\,1\ \ 1\ \ 0\ \ 0\,]^T$
4. bit : $[\,1\ \ 1\ \ 0\ \ 0\,]^T$